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الجامعة اللبنانية
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Entrance Exam 2007-2008 **Physics** **Duration: 2 hours**

I- [6 pts] Transient state – Steady state

A- (R, L) series circuit

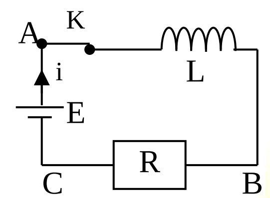


Fig. 1

In the circuit of figure 1, L = 1 H, R = 1 kΩ and E = 10 V.
At the instant $t_0 = 0$, we close the switch K. At the instant t, the circuit carries a current i.
An oscilloscope, conveniently connected, allows to display the variations of the voltage $u_R = u_{BC}$ as a function of time (Fig. 2).

1- Derive the differential equation describing the variations of the current i in terms of R, L, E and t.

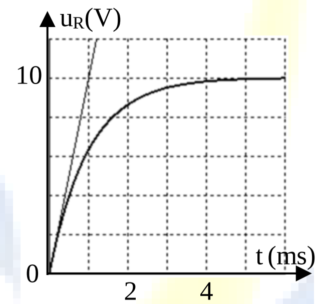


Fig. 2

2- The solution of this equation is of the form: $i = A_1 - B_1 e^{-\frac{t}{\tau_1}}$. Determine the values of the constants A_1, B_1 and τ_1 and give the physical significance of each.

3- Referring to figure 2, verify that the values of τ_1 and A_1 are equal to those calculated above.

4- Determine:

a) The duration t_1 at the end of which the steady state is practically reached;

b) The value of the energy stored in the coil starting from t_1.

B- A disk in a rotational motion

A disk can rotate about a horizontal axis (Δ) that is perpendicular to its plane through its center O. The moment of inertia I of the disk with respect to (Δ) is $I = 1.52\times10^{-5}$ kg·m^2.
When subjected to a motive couple, of constant moment $\mathcal{M}_M = 9.12\times10^{-3}$ m·N, the disk starts to rotate from rest at the instant $t_0 = 0$. At the instant t, the physical quantities θ and $\dot{\theta}$ are respectively the angular abscissa and the angular velocity of the disk. During its rotation, the disk undergoes also a braking couple of moment $\mathcal{M}_F = -k\dot{\theta}$, where k is a positive constant of value $k = 3.04\times10^{-5}$ SI units.
Applying the theorem of angular momentum, show that the differential equation in $\dot{\theta}$ that describes the motion of the disk is written as: $I\frac{d\dot{\theta}}{dt} + k\dot{\theta} = \mathcal{M}_M$.

C- An analogy

1- Match each of the physical electric quantities E, R, L, i, and $\frac{di}{dt}$ with the convenient mechanical physical quantity.

2- a) Determine the solution of the differential equation in $\dot{\theta}$.

b) Deduce the duration t_2 at the end of which the steady state is practically reached.

c) Determine the angular velocity in the steady state.

II- [7 pts] the iodine 131

Iodine 131 is one of the gaseous effluents that may escape from a nuclear reactor functioning with an enriched uranium. It is a β^- emitter of half-life T = 8.05 days, its daughter nucleus being the xenon (Xe). In fact, huge amounts of iodine 131 have been released during the accidents that occurred at Windscale (England) in 1957 (1.4×10^{15} Bq), at Three Mile Island (USA) in 1979 (5.5×10^{11} Bq) and at Chernobyl in 1986 (5×10^{17} Bq).

A- The fission of uranium 235

One of the possible fission reactions of uranium 235 yielding to iodine 131 is:

$${}^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{131}_{53}I + {}^{73}_{30}Zn + {}^{23}_{Z}F + y\,{}^{1}_{0}n .$$

Nucleus	Binding energy per nucleon (MeV)
U 235	7.59
I 131	8.42
Zn 73	8.64
F 23	7.62

1- Complete this equation.

2- Calculate the binding energy (E_ℓ) for each nucleus.

3- a) Show that the energy liberated by the reaction can be written in the form:

$E_{lib} = E_\ell\,(I) + E_\ell\,(Zn) + E_\ell\,(F) - E_\ell(U)$.

b) Calculate its value.

B- The disintegration of iodine 131

1. Write the disintegration reaction of the iodine 131.

2. a) Calculate the radioactive constant λ of the iodine.

b) Deduce the duration at the end of which the activity of the gaseous effluents released during the accident occurred at Chernobyl becomes equal to the initial activity that occurred during the accident at Three Mile Island.

3- Figure 3 shows the most probable disintegrations of iodine 131 into xenon 131.

a) What does Q represent?

b) i) Verify that the maximum kinetic energy of the emitted β_2 is 333 keV.

ii) Deduce the maximum kinetic energy of each of β_1 and β_3.

c) Calculate the maximum speed of each of β_2.

4. i) Calculate the energy of the photon γ_3 which is one of the most probable.

ii) This photon hits a metallic plate. An electron is extracted from this metal. Why?

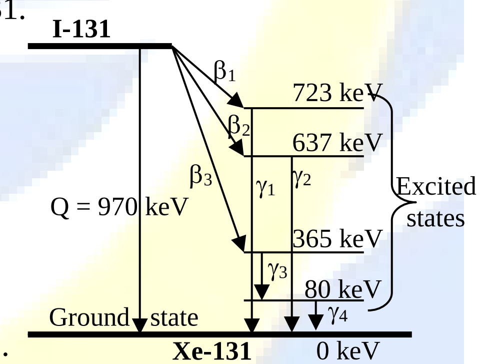


Fig. 3

Given: Mass of an electron: $m_0 = 511$ keV/c^2; $E_C = m_0c^2(\gamma-1)$ avec $\frac{1}{\gamma} = \sqrt{1-\frac{V^2}{c^2}}$.

III- [7 pts] Importance of an oscillating (L, C) circuit

A. Charging of the capacitor

In the circuit of the figure 4, C = 10 μF, L = 1 H, the value of R is adjustable and E = 10 V. An oscilloscope can display the variations of the voltage $u_C = u_{AM}$ and that of the voltage $u_R = u_{BM}$.

1- We adjust R to the value R = 50 Ω. At a given instant, the switch K is placed on the position (1).

a) Give the expression of the RC series circuit time constant τ.

b**)** Deduce the minimum duration at the end of which the capacitor can be considered practically charged.

2- Calculate, at the end of the charging, the energy stored by the capacitor.

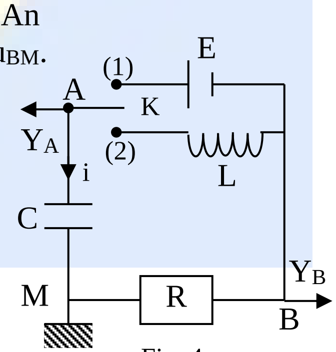


Fig. 4

B. Ideal oscillating circuit

We adjust R to the value zero and, at the instant $t_0 = 0$, we place K on the position 2.

1- Derive the differential equation describing the variations of u_C in terms of time.

2- Show that the time equation of u_C is: $u_C = A\cos(\frac{2\pi}{T_0}t)$. Calculate A and T_0.

C. Exploiting a waveform

We adjust R to the value R = 50 Ω. The oscilloscope gives us the curves of figure 5.

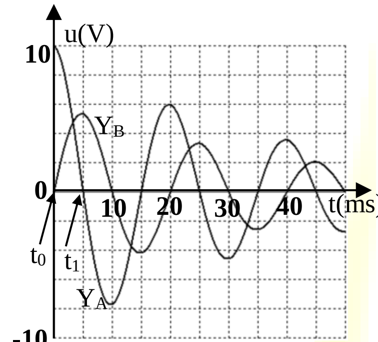
1- Calculate, at the instant $t_1 = 5$ ms:

a) The value of the current carried by the circuit;

b) The total energy stored in the circuit.

2- Deduce the average power lost between the instants t_0 and t_1.

3- Determine the duration T of one oscillation. Compare T with T_0.



$S_V(A) = 2$ V/div Fig. 5
$S_V(B) = 0.5$ V/div

D. The gantry

To avoid merchandise robbery, we attach to each item a small LC oscillating circuit. At the exit of a store, everyone is obliged to pass through a security gantry. This gantry always emits a radio wave of weak power and of frequency f = 10 MHz, that is exactly equal to the natural frequency f_0 of the small oscillator. In these conditions, the circuit picks-up the emitted energy, oscillates, and then emits a wave disturbing that emitted by the gantry. The detection of this disturbance triggers an alarm.

1-Why must f be equal to f_0?

2- The capacitance C' of the capacitor is equal to 0.5 nF. Determine the coil's inductance L'.

3- a) Calculate the wavelength of the radio wave emitted by the gantry (given : $c = 3.0\text{x}10^8$ m.s^{-1}).

b) This wave has to be emitted in many directions. It thus undergoes one of the physical phenomena: reflection, refraction or diffraction. Which one?

Entrance Exam 2007-2008 **Solution of Physics** **Duration: 2 hours**

I- [6 pts] Transient state – Steady state
A- (R, L) series circuit

1- We have $u_{AC} = E = L\frac{di}{dt} + Ri \Rightarrow \frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}$.

2- At $t_0 = 0$, $i = 0$, thus: $0 = A_1 - B_1$; $A_1 = B_1$. $\Rightarrow$: $i = A_1(1- e^{-\frac{t}{\tau_1}})$;

$\frac{di}{dt} = \frac{A_1}{\tau_1} e^{-\frac{t}{\tau_1}} \Rightarrow \frac{A_1}{\tau_1} e^{-\frac{t}{\tau_1}} + \frac{R}{L} A_1 - \frac{R}{L} A_1 e^{-\frac{t}{\tau_1}} = \frac{E}{L}$

By identification: $\tau_1 = \frac{L}{R} = 1$ ms, called time-constant of the circuit RL and $A_1 = B_1 = \frac{E}{R} = I_0 = 10$ mA, called value of the current in the steady state.

3- $\tau_1 = 1$ ms (tangent) ; $A_1 = B_1 = I_0 = u_R(max)/R = 10/10^3 = 10\times10^{-3} = 10$ mA

4- a) Duration $t_1 = 5\tau_1 = 5$ ms. b) $\mathcal{E} = ½ L I_0^2 = 5\times10^{-5}$ J

B- Disc in rotational motion

Theorem of angular momentum: $\sum M_\Delta = \frac{d\sigma_\Delta}{dt}$ avec $\sigma_\Delta = I\dot{\theta} \Rightarrow \mathcal{M}_M + \mathcal{M}_F + \mathcal{M}_\Delta(\vec{P}) + \mathcal{M}_\Delta(\vec{R}) = I\frac{d\dot{\theta}}{dt}$;

$I\frac{d\dot{\theta}}{dt} + k\dot{\theta} = \mathcal{M}_M$.

C- An analogy

1- By comparing the two equations, one will have: $\dot{\theta} \equiv i$; $\frac{d\dot{\theta}}{dt} \equiv \frac{di}{dt}$; $k \equiv R$; $L \equiv I$ and $\mathcal{M}_M \equiv E$.

2- a) By analogy with the solution in i : $\dot{\theta} = A_2 - B_2\, e^{-\frac{t}{\tau_2}}$.

For $t_0 = 0$, $\dot{\theta} = 0$, then: $A_2 = B_2 = \mathcal{M}_M/k = 300$ rd/s ; and $\tau_2 = \frac{I}{k} = 0{,}5$ s. $\dot{\theta} = 300(1 - e^{-\frac{t}{0,5}}) = 300(1 - e^{-2t})$

b) $t_1 = 5\tau_2 = 2.5$ s. c) Angular velocity (steady state) $\dot{\theta}_\ell = 300$ rd/s.

II- [7 pts] Iodine 131

A- The fission of uranium 235.

1. Conservation of mass number: $235 + 1 = 131 + 73 + 23 + y \Rightarrow y = 9$.

Conservation of charge number: $92 + 0 = 53 + 30 + z + 0 \Rightarrow z = 9$.

$^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{131}_{53}I + ^{73}_{30}Zn + ^{23}_{9}F + 9\ ^{1}_{0}n$.

2. For uranium: $E_\ell = 7.59\times235 = 1783.65$ MeV ; **for iodine:** $E_\ell = 8.42\times131 = 1103.02$ MeV .

for zinc: $E_\ell = 8.64\times73 = 630.72$ MeV ; **for fluorine:** $E_\ell = 7.62\times23 = 175.26$ MeV .

3. a) $E_{lib} = \{m(U) + m_n - [m(I) + m(Zn) + m(F) + 9m_n]\}\cdot c^2$.

But: $E_\ell(^{A}_{Z}X) = [Zm_P + Nm_n - m_X]\cdot c^2 \Rightarrow m_X = Zm_P + Nm_n - E_\ell/c^2$. $(N = A - Z)$

$m(U) = 92\ m_P + 143\ m_n - E_\ell(U)/c^2$; $m(I) = 53\ m_P + 78\ m_n - E_\ell(I)/c^2$;

$m(Zn) = 30\ m_P + 43\ m_n - E_\ell(Zn)/c^2$; $m(F) = 9\ m_P + 14\ m_n - E_\ell(F)/c^2$;

Thus: $E_{lib} = \{m(U) - [m(I) + m(Zn) + m(F) + 8\ m_n]\}\cdot c^2$

$= \{92 - (53 + 30 + 9)\}m_Pc^2 + \{143 - (78 + 43 + 14 + 8)\}m_nc^2 + \{E_\ell(I) + E_\ell(Zn) + E_\ell(F) - E_\ell(U)\}$

$E_{lib} = E_\ell(I) + E_\ell(Zn) + E_\ell(F) - E_\ell(U)$.

b) $E_{lib} = 1103.02 + 630.72 + 175.26 - 1783.65 = 125.35$ MeV

B- The disintegration of iodine 131

1. $^{131}_{53}I \longrightarrow ^{131}_{54}Xe + ^{0}_{-1}e + ^{0}_{0}\overline{\nu}$.

2. a) The radioactive constant $\lambda = \dfrac{\ln 2}{T} = 0.693/(8.05\times24\times3600) = 9.97\times10^{-7}\ s^{-1}$.

b) $A = A_0e^{-\lambda t}$, $e^{-\lambda t} = 5.5\times10^{11}/5\cdot10^{17} = 1.1\times10^{-6} \Rightarrow -9.97\times10^{-7}\ t = \ln(1.1\times10^{-6}) = -3.72$

That is to say $t = 1.38\times10^7\ s \approx 159.3$ days

3. a) Q represents the energy released by the disintegration of iodine 131.

b) i) $KE_{max}(\beta_2) = 970 - 637 = 333$ keV.

ii) $KE_{max}(\beta_1) = 970 - 723 = 247$ keV ; $KE_{max}(\beta_3) = 605$ keV.

c) The maximum speed of each one of β_2 : $333 = (511)(\gamma - 1)$; $\Rightarrow (\gamma - 1) = 0.652 \Rightarrow \gamma = 1.652$ and consequently:

$1 - \dfrac{V^2}{c^2} = 1/(1.652)^2 \Rightarrow V^2 = 0.633\ c^2$; thus $V = 0.796\ c = 2.388\times10^8$ m/s.

d) i- The energy of the photon $\gamma_3 = 285$ keV.

ii- Because the energy of the photon which is about 300 keV is much larger than the energy of extraction from the metal which must be of some eV. (- ½ for E> W)

III- [7 pts] Importance of the oscillating circuit (L, C)

A. Charging of the capacitor

1- a- The time-constant $\tau = RC$.

b) $\tau = 50\times10\times10^{-6} = 0.5$ ms ; $t = 5\tau = 2.5$ ms.

2- At the end of charging, the stored energy by the capacitor = $\frac{1}{2} CE^2 = 5\times10^{-4}$ J.

B. Ideal oscillating circuit

1- We have : $u_{AM} = u_C = -L\frac{di}{dt}$ avec $i = \frac{dq_A}{dt} = C\frac{du_C}{dt} \Rightarrow \frac{d^2u_C}{dt^2} + \frac{1}{LC}u_C = 0$.

2- The differential equation is of the form: $\ddot{x} + \omega_0^2 x = 0$; u_C is of the form $u_C = A\cos(\frac{2\pi}{T_0}t+\varphi)$; $\omega_0 = \frac{2\pi}{T_0}$

$\omega_0^2 = \frac{1}{LC} \Rightarrow T_0 = 2\pi\sqrt{LC} = 19.9$ ms .

At the instant $t_0 = 0$, $i = 0 \Rightarrow \varphi = 0$ and $A = 10$ V

C. Exploitation of a waveform

1. a) $u_R = 2.7\times0.5 = 1.35$ V and $i = u_R/50 = 0.027$ A.

b) The total energy stored in the circuit = $E_m + E_e = \frac{1}{2}Li^2 + \frac{1}{2}C u_C^2$

At the instant t_1, the total energy = $\frac{1}{2}\times1\times i^2 + 0 = 3.65\times10^{-4}$ J.

2. The average power lost between the instants t_0 and $t_1 = \frac{|\Delta E|}{t_1 - t_0} = (\frac{1}{2}10\times10^{-6}\times100 - 3.65\times10^{-4})/5\times10^{-3} = 0.027$ W.

3. $T = 20$ ms, $T \approx T_0$

D. The gantry

1- So that the circuit collects emitted energy it is necessary that it is adjusted to the frequency of the transmitter (electric resonance - selective circuit $\Leftrightarrow$ the phenomenon of resonance).

2- $T_0' = 10^{-7} = 2\pi\sqrt{L'C'} \Rightarrow L' = 0.5\mu$H.

3- a) $\lambda = c/f = 30$ m.

b) Phenomenon of diffraction.