LEBANESE UNIVERSITY
FACULTY OF ENGINEERING

الجامعة اللبنانية
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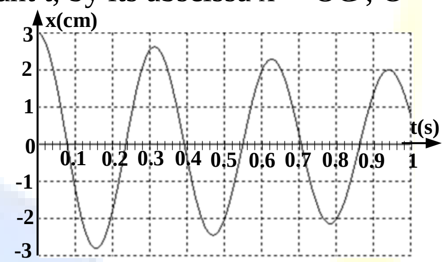
Entrance exam 2006-2007 **Duration: 2 hours**

Physics

I- [21 pts] An oscillator

A mechanical oscillator (C) is formed of a solid (S), of mass m, attached to the extremity A of a horizontal spring of stiffness (constant) k = 80 N/m whose other extremity B is fixed. The solid (S) can move on a horizontal rail. The position of its center of gravity G is located, at an instant t, by its abscissa $x = \overline{OG}$, O being its equilibrium position.



A) Theoretical study

We neglect all frictional forces.

1. Derive the differential equation that describes the oscillations of (C).
2. Determine the expression of the natural period T_0 of these oscillations.

B) Experimental study

1- Free oscillations

An appropriate device, connected to a computer, gives us the curve of the above figure representing the variation of x in terms of time.

a. Determine, from the graph, the duration T of one oscillation of the solid.
b. Determine the average power dissipated between the instants 0 and 3T.

2- Forced oscillations

Now, the extremity B is connected to a vibrator of adjustable frequency f and for each frequency f a recording is performed. The table below shows the amplitude X_m relative to each f.

a. Using the table, determine, with justification, an approximate value of T_0.
b. Determiner then the value of m.
c. What do we obtain:
  i. in the absence of all frictional forces?
  ii. in the case where the magnitude of the frictional force is increased?

f (Hz)	1.5	2.0	2.5	2.8	3.0	3.2	3.3	3.6	4.0	4.5
X_m (cm)	0.4	0.6	1.0	1.5	2.1	2.3	2.0	1.5	1.0	0.7

C) The hydrogen chloride molecule

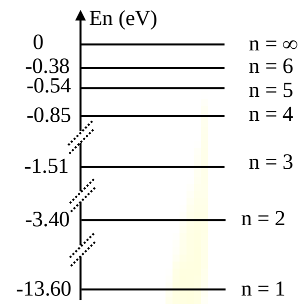
A hydrogen chloride molecule (HCl) can be represented by a harmonic oscillator of mass $m_H = 1.67\times10^{-27}$ kg, and of stiffness k'. The potential energy due to the interaction between the two atoms can be reduced to:

$E_P(x) = \dfrac{0.27e^2}{4\pi\varepsilon_0 r_0^3}x^2$; where: $\dfrac{1}{4\pi\varepsilon_0} = 9\times10^9$ SI; $e = 1.60\times10^{-19}$ C; and $r_0 = 1.3\times10^{-10}$ m ; r_0 being the distance between the two atoms at equilibrium and x is the displacement of the hydrogen atom with respect to its equilibrium position with $x \ll r_0$.

This molecule, when excited with an electromagnetic wave of frequency ν, oscillates with a maximum amplitude X_m where $X_m \ll r_0$. Determine, with justification, the value of ν.

II- [18 pts] Hydrogen atom and diffraction

A- Hydrogen atom



The adjacent figure shows some energetic levels of a hydrogen atom.

1. Determine, with justification, the behavior of a hydrogen atom taken at the fundamental state when it receives a photon of energy: a) 12.75 eV; b) 10.99 eV and c) 15.61 eV.

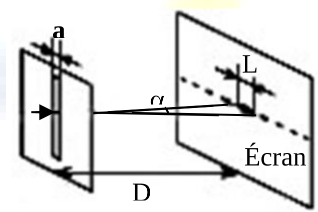
2. The downward transition of a hydrogen atom from an excited level (n>1) to the fundamental state gives the Lyman series. Calculate the shortest and longest wavelengths of the extreme radiations corresponding to this series.

3. The downward transition of a hydrogen atom from an excited level (n > 2) to the first excited level gives the Balmer series. Calculate the longest wavelength of the radiation corresponding to this series.

4. Specify, with justification, the radiation that belongs to the visible spectrum.

Given: $h = 6.626 \times 10^{-34}$ *J.s;* $c = 2.998 \times 10^{8}$ *m/s; 1 eV =* 1.602×10^{-19} *J.*

B- Diffraction



A diffraction experiment is performed with a light emitted by a hydrogen lamp provided with four filters. Each filter allows the passage of a monochromatic radiation. The hydrogen lamp is placed in front of a slit of width a = 0.5 mm. For each of these four radiations, a diffraction pattern is observed on a screen placed at a distance D = 1.600 m from the slit.

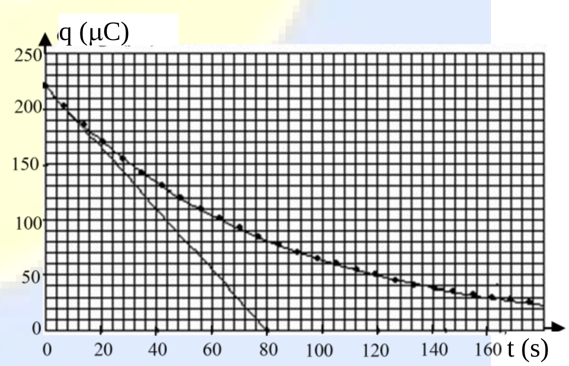
The measurement of the width L of the central fringe gives, for the used radiations, the respective values: $L_1 = 4.20$ mm ; $L_2 = 3.11$ mm ; $L_3 = 2.78$ mm and $L_4 = 2.63$ mm. Let α be the angular width of the central fringe (see adjacent figure).

1. Determine the wavelengths of the used four monochromatic radiations.

2. Determine, with justification, the transition corresponding to the emission of each of these radiations.

III- [21 pts] An analogy

First part: Exponential decay of the charge of a capacitor



A capacitor (C), of capacitance C = 0.220 mF and of initial charge Q_0 , is put in series in a circuit formed of a resistor (R) of resistance R, a switch (K) and connecting wires.

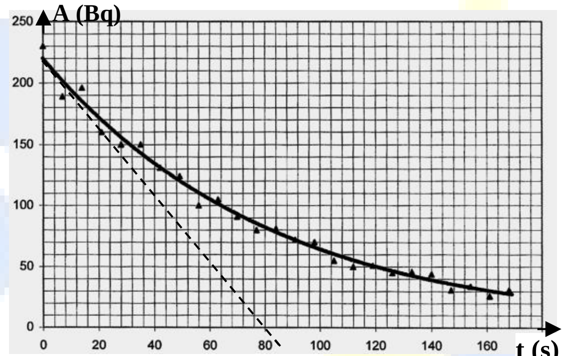
We close (K) at the instant $t_o = 0$. At the instant t, (C) has the charge q (q>0) and the circuit carries the current i. An appropriate device allows us to obtain the variation of q in terms of time (graph of the adjacent figure)

1. Make a diagram of the circuit showing the **real direction** of the current and specifying the armature that has the charge q.
2. Derive the differential equation describing the evolution of q in terms of time.
3. The solution of this equation is of the form $q = A_1 + B_1 e^{-\alpha t}$ where A_1, B_1 and α are constants.
 a. Determine A_1, B_1 and $1/\alpha$ and specify the meaning of $1/\alpha$.
 b. Write down, in terms of R, C and t, the expression giving the number N_e of the electrons on the armature that has an excess of electrons.
4. Use the adjacent figure to find the value of $1/\alpha$. Deduce the value of R.
5. Determine the relation between i and N_e.
6. Determine the energy delivered by the capacitor between the instants $t_0 = 0$ and $t_1 = 1/\alpha$.

Second part: Exponential decay of the radon 220

In an experimental session, we study the radioactive decay of the activity A of a sample of radon 220 ($^{220}_{86}Rn$).

The adjacent figure shows the shape of the curve giving the variation of A in terms of time.



1. A radon nucleus, an α emitter, disintegrates into a polonium nucleus (Po). Write down the equation of this disintegration.
2. The instantaneous expression of the activity A is given by: $A = A_0 e^{-\lambda t}$, where λ is the radioactive constant of the sample.
a. i. Define the activity of a radioactive sample of a substance and give the relation between A and dN/dt, where N represents the average number of the radon nuclei that are present at the instant t
ii. Determine, with justification, the physical quantity in the first part (A) which is analogous to the activity in the second part.
b. How can we determine, from the graph, the value of $1/\lambda$? Determine its value.
3. After what time can we consider that the sample is practically completely disintegrated?
4. By comparing the two figures, show that the radioactivity has a hazardous character.
5. Determine the energy liberated by the radon sample between the instants $t_0 = 0$ and $t_1 = 1/\lambda$.
Given: m(Rn) = 220.011384 u; m(Po) = 216.001905 u; m(α) = 4.002603 u; 1 u = 931.5 MeV/c^2.

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Solution of Physics

I- An oscillator

A) Theoretical study

1. No friction, conservation of ME(C): ME = KE + EPE = ½ mV^2 + ½ kx^2 = constant.

Let us derive with respect to time: $mV\dot{V} + kx\dot{x} = 0$; $\ddot{x} + \frac{k}{m}x = 0$

2. This equation is of the form $\ddot{x} + \omega_0^2 x = 0$, $\Rightarrow \omega_0 = \sqrt{\frac{k}{m}}$ et $T_0 = \frac{2\pi}{\omega_0}$, then $T_0 = 2\pi\sqrt{\frac{m}{k}}$

B) Experimental study

1- Free oscillations

a. We have 3T = 0.94 s $\Rightarrow$ T = 0.94/3 = 0.313 s.

b. $P_{av} = \frac{\Delta ME}{\Delta t}$; ΔME = ; ΔPE(maximum) = ½ k ($X^2_{m\,final} - X^2_{m\,initial}$) = ½ 80(9 – 4)×10^{-4} = 2×10^{-2} J

And Δt = 3T = 0.94 s. Thus : P_{av} = 2×10^{-2}/0.94 = 2.13×10^{-2} W.

2. **Forced oscillations**

a. According to the table, the amplitude of the oscillations takes a maximum value (resonance of amplitude) when the frequency f of the excitations is equal to f = 3.2 Hz. According to the graph, (free oscillations) the oscillations are slightly small, then $f \approx f_0 \Rightarrow T_0 = T = 1/f = 1/3.2 = 0.3125 \approx 0.313$ s

b. $T_0 = 2\pi\sqrt{\frac{m}{k}}$; $m = \frac{kT_0^2}{4\pi^2}$ = 80×$0.313^2/4\pi^2$ = 0.199 kg

c. i. In the absence of any force of friction, the amplitude X_m passes by a very large value (∞) for $T = T_0$ and there is risk of rupture of the spring.

ii. When we increase the magnitude of the forces of friction, the amplitude X_m decrease and the pseudo-period of resonance of amplitude is larger than T_0. The resonance, which was acute, becomes less and less acute to become fuzzy. (As long as the critical mode yet was not reached).

C) The hydrogen chloride molecule

Determination of k' : PE(x) = ½ $k'x^2 = \frac{0.27e^2}{4\pi\varepsilon_0 r_0^3}x^2 \Rightarrow k' = \frac{2\times0.27\times1.6^2\times10^{-38}\times9\times10^9}{1.3^3\times10^{-30}}$ = 56.63 N/m

The proper frequency of oscillations of the molecule is: $\nu_0 = \frac{1}{2\pi}\sqrt{\frac{k'}{m}} \Rightarrow \nu_0 = \frac{1}{2\pi}\sqrt{\frac{56.63}{1.67\times10^{-27}}}$

$\Rightarrow \nu_0 = 2{,}93\times10^{13}$ Hz. Thus, the resonance of amplitude takes place for $\nu = \nu_0 = 2.93\times10^{13}$ Hz

II- Hydrogen atom and diffraction

A- Hydrogen atom

1. a) An energy gain of 12.75 eV would lead the hydrogen atom to an energy of:
$-13.60 + 12.75 = -0.85$ eV.
This energy is that of the level $n = 4$. The photon is well absorbed, the atom passes to the level $n = 4$.
b) An energy gain of 10.99 eV would lead the hydrogen atom to an energy of:
$-13.60 + 10.99 = -2.61$ eV. This value does not correspond to any energy level of the hydrogen atom. This atom thus remains at the fundamental level, the photon in question is not absorbed.
c) This contribution of energy (15.61 eV) exceed the energy of ionization (13,60 eV). The atom is thus ionized and the free electron leaves with a kinetic energy:
$-13.60 + 15.61 = 2.01$ eV.

2. a- The smallest energy emitted by the hydrogen atom corresponds in the passage of the excited level $n = 2$ ($E_2 = -3.39$ eV) to the fundamental level ($E_1 = -13{,}60$ eV). The emitted energy is thus:
$E_{21} = E_2 - E_1 = -3.39 - (-13.60) = 10.21 \text{ eV} = 10.21\times1.602\times10^{-19} \text{ J} = 1.636\times10^{-18}$ J.
The wave associated with the emitted photon has a frequency ν_{21} and a wavelength λ_{21} satisfying :
$E_{21} = h\,\nu_{21} = h.c / \lambda_{21}$
$\lambda_{21} = h.c/E_{21} = 6.626\times10^{-34}\times2.998\times10^{8}/(1.636\times10^{-18})$.
$\lambda_{21} = 1.214\times10^{-7}$ m = 121.4 nm.

b- The greatest energy emitted by the hydrogen atom corresponds in the passage from the maximum energy level (Emax = 0 eV) at the fundamental level ($E_1 = -13{,}60$ eV). The emitted energy is thus:
$\Delta E_{max1} = 13.60 \text{ eV} = 13.60\times1.602\times10^{-19} \text{ J} = 2.179\times10^{-18}$ J.
the wave associated with the emitted photon has a wavelength λ_{max1} satisfying:
$\lambda_{max1} = h.c/\Delta E_{max1} = 6.626\times10^{-34}\times2.998\times10^{8}/(2.179\times10^{-18})$;
$\lambda_{max1} = 9{,}12\times10^{-8}$ m = 91,2 nm.

3. – The largest wavelength thus corresponds to the emission of a photon having the smallest energy which corresponds in the passage from the level $n = 3$ to the level $n = 2$:
$\Delta E_{32} = 1.89 \text{ eV} = 1.89\times1.602\times10^{-19} \text{ J} = 3.028\times10^{-19}$ J.
The wave associated with the emitted photon has a wavelength:
$\lambda_{32} = h.c/\Delta E_{32} = 6.626\times10^{-34}\times2.998\times10^{8}/(3.028\times10^{-19})$;
$\lambda_{32} = 6.56\ 10^{-7}$ m = 656 nm.
4- The radiation of wavelength λ_{32} is visible, because its wavelength in vacuum is between 400 nm and 800 nm. While the others are invisible (ultraviolet)

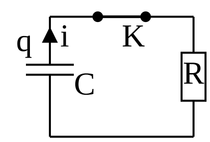
B- Diffraction

1.. We know that $\alpha = 2\theta$ with $\theta = \lambda/a$. According to the figure $\tan\alpha = L/D = \alpha$, because L/D is very weak.
But $\alpha = 2\theta$; thus: $L/D = 2\lambda/a \Rightarrow \lambda = La/2D$. By applying this relation, we obtain:
$\lambda_1 = 4.2\times10^{-3}\times0.5\times10^{-3}/2\times1.6 = 6.56\times10^{-7}$ m ; $\lambda_2 = 4.86\times10^{-7}$ m; $\lambda_3 = 4.34\times10^{-7}$ m and $\lambda_4 = 4.11\times10^{-7}$ m;

2. They belong to the Balmer series because they are visible (400 nm < λ < 800 nm). The 4 transitions must correspond to the passage of the excited levels n = 3, 4, 5 and 6 to the excited level n = 2 :
λ_1 from n = 3 to n = 2; λ_2 from n = 4 to n = 2; λ_3 from n = 5 to n = 2 and λ_4 from n = 6 to n = 2. (Or by making calculation)

III- An analogy

A- Exponential decay of the charge of a capacitor



1. See adjacent figure.

2. We have $u_C = Ri$. But $u_C = \frac{q}{C}$ and $i = -\frac{dq}{dt}$, thus: $\frac{q}{C} = -R\frac{dq}{dt}$.

Finally: $\frac{dq}{dt} + \frac{1}{RC}q = 0$.

3. **a**. For $t_0 = 0$: $Q_0 = A_1 + B_1$ and for $t = \infty$, $q = 0 = A_1 \Rightarrow A_1 = 0$ and $B_1 = Q_0$;
thus $q = B_1e^{-\alpha t}$ et $\frac{dq}{dt} = -\alpha B_1e^{-\alpha t} \Rightarrow -\alpha B_1e^{-\alpha t} + \frac{1}{RC}B_1e^{-\alpha t} = 0 \Rightarrow \alpha = \frac{1}{RC}$ and $1/\alpha = RC$.

b. The armature which carries an excess of electrons at the date t carries the charge –q ; thus:
$N_e = -q/(-e) = q/e$; (with $Q_0 = 2.20\times10^{-4}$ C)

$N_e = \frac{Q_0}{e}e^{-\frac{t}{RC}} \Rightarrow N_e = 1.375\times10^{15}e^{-\frac{t}{RC}}$ electrons

4. On the graph $1/\alpha = \tau = 80$ s (point of meeting of the tangent at the origin with the asymptote).
$RC = \tau = 80$ s $\Rightarrow R = 80/0.22\times10^{-3} = 3.640\times10^5\ \Omega$,

5. The instantaneous expression of i : $i = -\frac{dq}{dt} = -e\frac{dN_e}{dt} \Rightarrow \frac{i}{e} = -\frac{dN_e}{dt}$.

6. The energy provided by the capacitor $W = W_0 - W\tau = \frac{1}{2}\frac{Q_0^2}{C} - \frac{1}{2}\frac{q^2}{C}$ with $q = 0.37Q_0 = 80\times10^{-6}$C

$W = \frac{1}{2\times0.22\times10^{-3}}[(220\times10^{-6})^2 - (80\times10^{-6})^2] = 9.55\times10^{-5}$ J

B- **Exponential decay of radon 220**

1. $^{220}_{86}Rn \rightarrow ^{216}_{84}Po + ^{4}_{2}He$

2. a i. Average number of disintegrations per unit of time: A = dN/dt

ii. $A = -\frac{dN}{dt}$ is equivalent to the current i divided by e : $\frac{i}{e} = -\frac{dN_e}{dt}$;

Thus A is equivalent to $\frac{i}{e}$.

b. λ plays the same role as α in the first part. Donc $1/\lambda$ is the intersection of the tangent to the curve at $t_0 = 0$ with the axis of time. $1/\lambda = 80$ s and $\lambda = 0.0125\ s^{-1}$.

3. By equivalence with the first part, we can say that $5(1/\lambda) = 400$ s

4. For disintegration, the points are distributed around the curve; thus its character is a random character.

5. At the end of t_1, the number of disintegrated nuclei = $N_0 - N = 1/\lambda[\ A_0(1- e^{-\lambda t})] = 80[220(1-e^{-1})$

$N_0 - N = 1.11\times10^4$ nuclei.

The energy released by the disintegration of a nucleus: $E_1 = \Delta m\ c^2$

$\Delta m = (220.011384 - 216.001905 - 4.002603) = 6.87\times10^{-3}$ u

$E_1 = 6.87\times10^{-3} \times 931.5 = 6.399$ MeV.

The total energy released between $t_0 = 0$ and $t_1 = 1/\lambda = (N_0 - N)\ E_1 = 1.11\times10^4\times6.399 = 71033$ MeV.