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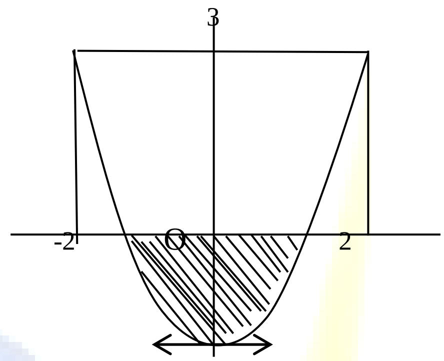
Entrance Exam 2005 -2006 **Mathematics** **Duration : 3 hours**
16/07/2005

The distribution of grades is over 25

I-(2.5pts) consider the integral $I_n = \int_0^1 \frac{x^n}{x^2+1}dx$ where n is a natural number.



1-Prove that, for all n, $0 \le I_n \le \frac{1}{n+1}$. Deduce $\lim\limits_{n\to\infty} I_n$

2-Calculate I_0 and I_1

3-a) Prove that, for all n, $I_{n+2} + I_n = \frac{1}{n+1}$

b) Deduce that, $\int_0^1 \frac{x^4}{x^2+1}dx = \frac{\pi}{4} - \frac{2}{3}$

4-The adjacent figure shows the representative curve of the function f defined on [-2; 2] by $f(x) = \frac{x^4-1}{x^2+1}$ <u>using the previous results,</u> calculate the area of the shaded domain.

II- (4 pts) **<u>A-</u>** Let (U_n) be the sequence defined for $n \ge 2$ by $U_n = 2^{n-1} - (n+1)$

1-a) Calculate the first three terms of the sequence (U_n)

b) Prove that (U_n) is strictly increasing.

2- Knowing that $\lim\limits_{n\to\infty} \frac{2^x}{x} = +\infty$, prove that the sequence (U_n) is divergent

<u>B-</u> From an urn containing 10 white balls and 10 black balls, we draw one ball and then we put it back in the urn (drawing with replacement). We execute this trial n times ($n \ge 2$).

Consider the events ***C, D, E, F*** and ***G*** defined by:

C : « the drawn balls are black »

D : « the drawn balls have the same color »

E : « the drawn balls are not of the same color »

F : « among the drawn balls, only one is white »

G : « among the drawn balls, at most one is white»

1- Calculate the probability of each of the events **C** and D

2-a) Prove that $p(E) = 1 - \frac{1}{2^{n-1}}$, $p(F) = \frac{n}{2^n}$ et $p(G) = \frac{n+1}{2^n}$

b- Prove that **E** $\cap$ **G** = **F**. Deduce that the events **E** and **G** are independent if and only if $2^{n-1} = n+1$

3-Using part A-1, Prove that the events **E** and **G** are independent <u>for only one</u> value of n to be determined.

III- (6 pts) The plane is referred to a direct orthonormal system $(O\,;\,\vec{u}\,,\,\vec{v})$

Consider the ellipse (E) of focus O, directrix the straight line (d) of equation $x=\frac{5}{2}$ and eccentricity $\frac{2}{3}$

1-a) Write an equation of (E) and determine its center

b) Prove that the point F(-4; 0) is the second focus of (E) and the straight line (δ) of equation $x=-\frac{13}{2}$ is the associated directrix.

c) Determine the points of intersection P and Q of (E) and the y-axis and draw (E)

2-Let M be a variable point of affix $z = re^{i\theta}, 0<\theta<\pi,$ belonging to (E)

a) Calculate the distance of M from the directrix (d) in terms of r and θ

b) Deduce that $OM = \dfrac{5}{3+2\cos\theta}$

3-a) The straight line (OM) cuts (E) again at a point M´. Prove that $OM' = \dfrac{5}{3-2\cos\theta}$

b) Determine θ so that the length MM´ is minimum.

4- Let N_1 (x_1 ; y_1) be any point of (E) such that $y_1 \neq 0$.

a) Determine an equation of the tangent (T_1) to (E) at N_1 and prove that (T_1) cuts the directix (δ) At the point of ordinate $\dfrac{5(x_1+4)}{2y_1}$

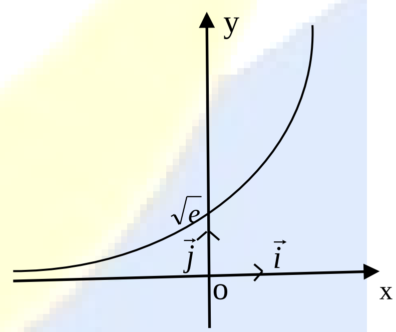
b) Let N_1 (x_2; y_2) be the point of (E) such that N_1, $N_2$and F are collinear.
Prove that $x_1y_2-x_2y_1=4(y_1-y_2)$. Deduce that the tangents to (E) at N_1 and N_2 intersect on (δ)

c) When θ varies, on what straight line do the tangents to (E) at M and M′ intersect? justify.

IV- (6.5 pts) **A-**Consider the differential equation (E) : $2y' - y = 0$

1-Solve the equation (E)

2-The adjacent figure shows the representative curve of a function g which is a particular solution of the equation (E).



a) Determine the function g.

b) Prove that g has an inverse function g^{-1} whose domain of definition is to be determined.

c) Draw the representative curve (γ) of g^{-1} and prove that $g^{-1}(x) = 2\ln x - 1$

B- Consider the function f defined on $]0;+\infty[$ by $f(x) = |\ln x| + \left|\ln \frac{x}{e}\right|$

Let (C) be the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$

1-a) Prove that $f(x) = \begin{cases} 1-2\ln x & \text{if } x \in\ ]0;1[\\ 1 & \text{if } x \in [1;e] \\ 2\ln x - 1 & \text{if } x \in\ ]e;+\infty[\end{cases}$

b) Prove that f is continuous at 1 and at e

2-a) Calculate $f(\frac{1}{2})$ and $f(e^2)$

b) Using the curve (γ) drawn in part A, construct the curve (C).

3- Let a and b two strictly positive numbers such that $ab=e$. Prove that $f(a) = f(b)$

4-a) Prove that, for all $\lambda > 1$. The equation $f(x) = \lambda$ admits two roots x_1 and x_2 such that $x_1x_2 = e$

b) Determine the solutions of each of the equations $f(x) = 1+\ln 4$ and $f(x) = 3$

5-Let (P) be the curve of equation $y = e^{f(x)}$ where $x \geq e$. Prove that (P) is a part of a parabola whose focus and directrix are to be determined.

V-(6 pts) Given a right triangle ABC such that $(\overrightarrow{AB};\overrightarrow{AC}) = \frac{\pi}{2}\ (2\pi)$;

Let H be the orthogonal projection of A on (BC)

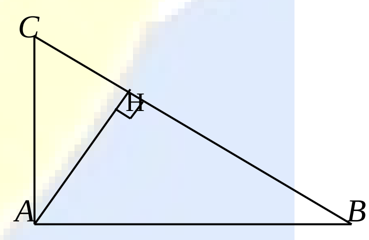
1-Let h be the dilation (homothecy) of center H that transforms C onto B
Determine and construct $h(AC)$. Deduce the image D of A under h.

2-Let S be the similitude that transforms A onto B and C onto A

a) Determine the angle of S.

b) Determine the image by S of each of the two straight lines (AH) and (CH)

c) Deduce that H is the center of S.



3-a) Determine the image by S of each of the two straight lines (AB) et (CB)

b) Prove that $S(B) = D$ and deduce that $S\circ S(A) = h(A)$

c) Prove that $S\circ S = h$

4- Let E be the midpoint of $[AC]$

a) Determine the points $F = S(E)$ and $G = S(F)$

b) Prove that the points E, H, and G are collinear and that the triangle EFG is right.

5-Suppose in this part that AB=6 and AC=4, and that the plan is referred to a direct orthonormal

system $(A, \overrightarrow{u}, \overrightarrow{v})$ such that $\overrightarrow{u} = \frac{1}{6}\overrightarrow{AB}$

a) Determine the complex form of S. Deduce the ratio of S and that of h

b) Deduce the coordinates of each of H and D.

Entrance Exam 2005 -2006 **Solution of Mathematics** **Duration : 3 hours**
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1) $\dfrac{x^n}{1+x^2} \geq 0$ for $0 \leq x \leq 1 \Rightarrow I_n \geq 0$

$\dfrac{x^n}{1+x^2} \leq x^n$ then $\int_0^1 \dfrac{x^n}{1+x^2}dx \leq \int_0^1 x^n dx$ *where* $\int_0^1 \dfrac{x^n}{1+x^2}dx \leq \left[\dfrac{x^{n+1}}{n+1}\right]_0^1$

$I_n \leq \dfrac{1}{1+n}$, hence $0 \leq I_n \leq \dfrac{1}{1+n}$

$\lim\limits_{n\to\infty}\dfrac{1}{1+n} = 0$, therefore $\lim\limits_{n\to\infty} I_n = 0$

2) $I_0 = \int_0^1 \dfrac{dx}{x^2+1} = [\arctan x]_0^1 = \dfrac{\pi}{4}$

$I_1 = \int_0^1 \dfrac{x}{x^2+1}\,dx = \dfrac{1}{2}[\ln(x^2+1)]_0^1 = \dfrac{1}{2}\ln 2$

3) a- $I_{n+2} + I_n = \int_0^1 \dfrac{x^{n+2}}{x^2+1}\,dx + \int_0^1 \dfrac{x^n}{x^2+1}\,dx = \int_0^1 \dfrac{x^{n+2}+x^n}{x^2+1}\,dx = \int_0^1 \dfrac{x^n(x^2+1)}{x^2+1}\,dx = \int_0^1 x^n dx = \dfrac{1}{n+1}[x^{n+1}]_0^1 = \dfrac{1}{n+1}$

b- $\int_0^1 \dfrac{x^4}{1+x^2}\,dx = I_4$ but $I_0 + I_2 = I \Rightarrow I_2 = 1 - \dfrac{\pi}{4}$

then $I_2 + I_4 = \dfrac{1}{3} \Rightarrow I_4 = \dfrac{1}{3} - 1 + \dfrac{\pi}{4} = \dfrac{\pi}{4} - \dfrac{2}{3}$

4) $f(x) = 0$; *then* $x = -1$ *or* $x = 1$

f is an even function , then the area of the shaded region:

A=-2$\int_0^1 f(x)dx = 2(I_0 - I_4) = \dfrac{4}{3}$ square units

II- A)

1-a) The three term of the sequence U_n are : $U_2 = 2^{2-1} - (2+1) = -1$; $U_3 = 2^{3-1} - (3+1) = 0$;
$U_4 = 2^{4-1} - (4+1) = 3$

b) $U_{n+1} - U_n = 2^n - (n+2) - 2^{n-1} + (n+1) = 2^{n-1} - 1 > 0$ because $n \geq 2$ so
the sequence (U_n) is strictly increasing

2- $\lim_{n\to\infty} U_n = \lim_{n\to\infty}(\frac{1}{2}\times 2^n - n - 1) = \lim_{n\to\infty} n(\frac{1}{2}\times\frac{2^n}{n} - 1 - \frac{1}{n}) = (+\infty)\times(+\infty) = +\infty$

Then the sequence (U_n) is divergent.

B) 1-The probability of getting a black ball is $\frac{10}{20} = \frac{1}{2}$

Getting n black balls is the same as getting 1 black ball in n independent drawings, so
p(C)=*p(N)* x *p(N)* x…x *p(N), n* time

where : $p(C) = \frac{1}{2}\times\frac{1}{2}\times.........\frac{1}{2}, n\ times$

hence $p(C) = (\frac{1}{2})^n = \frac{1}{2^n}$

$p(D) = p(n$ black balls or n white balls$)$
$= p(n$ black balls$) + p(n$ white balls$)$

But $p(n$ white balls$) = \frac{1}{2^n}$ *so* $p(D) = \frac{1}{2^n} + \frac{1}{2^n} = \frac{2}{2^n} = \frac{1}{2^{n-1}}$

2-a) E and D are two opposite events, then $P(E) = 1 - P(D)$ hence $P(E) = 1 - \frac{1}{2^{n-1}}$

F is the set of n-uplets containing each 1 white balls and n-1 black balls and since there are n possible places for the white ball then : $P(F) = n\times\frac{1}{2}\times(\frac{1}{2})^{n-1} = n.\frac{1}{2^n} = \frac{n}{2^n}$

$P(G) = P(F\cup C) = P(F) + P(C)$ (C and F are incompatible)

$\Rightarrow P(G) = \frac{n}{2^n} + \frac{1}{2^n} = \frac{n+1}{2^n}$

b) E is the events
Getting (1w and n-1 B) or (2w and n-2 B) or (n-1 w and 1 B)
G : is the events
Getting (0w and n B) or (1w and n-1 B)
Then $E\cap G$ is the event of getting (1w and n-1 B) consequently $E\cap G$ = F
E and G are independent if and only if
$p(E\cap G) = p(E) \times p(G)$, $p(E) \times p(G) = p(F)$ which gives that

$\frac{n}{2^n} = \left(1 - \frac{1}{2^{n-1}}\right) . \frac{n+1}{2^n}$ so $n = n + 1 - \frac{n+1}{2^{n-1}}$ or $\frac{n+1}{2^{n-1}} = 1$

Then $2^{n-1} = n+1$

3- E and G are independent then $2^{n-1} = n+1$, $U_n = 0$

But (U_n) is strictly increasing and $U_3 = 0$ so there exist only one value for n which is 3.

III)

1) a- Let M(x, y) be a variable point of (E) is orthogonal projection on (d) is

$M'(\frac{5}{2}; y)$ then, $\frac{MO}{MM'} = e = \frac{2}{3}$

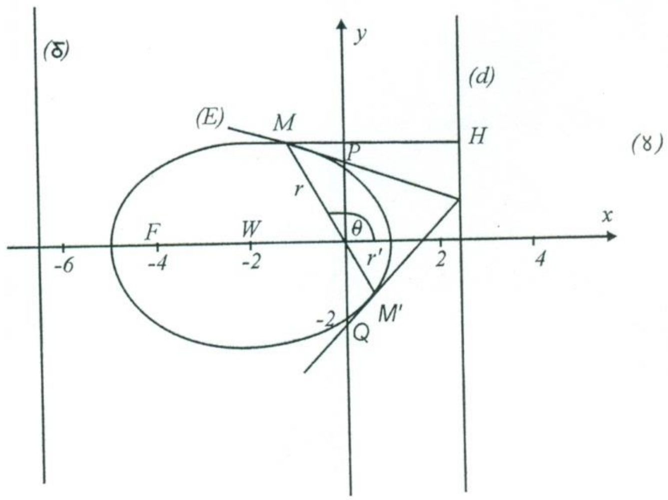
$9(x^2 + y^2) = 4(x - \frac{5}{2})^2$, Let $5x^2 + 9y^2 + 20x - 25 = 0$ an equation equivalent $\frac{(x+2)^2}{9} + \frac{y^2}{5} = 1$

The center of (E) is the point w(-2;0)

b- The point F is the symmetric of O with respect to w then it is the point F(-4, 0) , the associated directrix of F is the symmetric of (d) with respect to w, then it is the straight line (δ) of equation $x = \frac{-13}{2}$

c- For x = 0 we have $\frac{4}{9} + \frac{y^2}{5} = 1$ which gives $\frac{y^2}{5} = 1 - \frac{4}{9} = \frac{5}{9}$

then $y^2 = \frac{25}{9} \Rightarrow y = \frac{5}{3}$ *or* $y = -\frac{5}{3} \Rightarrow P(0, \frac{5}{3})$ *and* $Q(0, -\frac{5}{3})$ or conversely.



2) a- $d(\mathrm{M};(d)) = MH = \left|x_M - \frac{5}{2}\right| = \left|r\cos\theta - \frac{5}{2}\right|$

b- $OM = r$ and $x_M < \frac{5}{2}$ then $r\cos\theta - \frac{5}{2} < 0$ where $d\,(\mathrm{M};(d)) = \frac{5}{2} - r\cos\theta$

But $\frac{OM}{MH} = e = \frac{2}{3} \Rightarrow r = \frac{2}{3}\left(\frac{5}{2} - r\cos\theta\right)$

Where r = OM = $\frac{5}{3+2\cos\theta}$

3) a- $z_{M'} = z' = r'e^{i\theta'}$ with $\theta' = (\vec{u};\overrightarrow{OM}) = \pi + \theta(2\pi)$

$$\Rightarrow OM' = r' = \frac{5}{3+2\cos\theta'} = \frac{5}{3+2\cos(\pi+\theta)} = \frac{5}{3-2\cos\theta}$$

b- $MM' = MO + OM' == \frac{30}{9-4\cos^2\theta}$

For MM′ to be minimum $9-4\cos^2\theta$ should be maximum

so $9-4\cos^2\theta \le 9$ then MM' is minimum when

$9-4\cos^2\theta = 9 \Rightarrow \cos^2\theta = 0 \Rightarrow \cos\theta = 0$ and $0<\theta<\pi$ then $\theta = \frac{\pi}{2}$

4) a - $\frac{(x+2)^2}{9} + \frac{y^2}{5} = 1 \Rightarrow \frac{2}{9}(x+2) + \frac{2yy'}{5} = 0$ at point $N_1(x_1\,;\,y_1)$,

$\Rightarrow y'_{N_1} = \frac{-5}{9}.\frac{x_1+2}{y_1}$ an equation of the tangent (T_1) is $y - y_1 = \frac{-5(x_1+2)}{9y_1}(x - x_1)$

(T_1) : $y - y_1 = \dfrac{-5(x_1+2)}{9y_1}(-\dfrac{13}{2} - x_1)$

$\Rightarrow y = \dfrac{65(x_1+2)}{18y_1} + \dfrac{5x_1(x_1+2)}{9y_1} + y_1 = \dfrac{10x_1^2 + 18y_1^2 + 85x_1 + 130}{18y_1}$

Or $5x_1^2 + 9y_1^2 + 20x_1 - 25 = 0 \Rightarrow 5x_1^2 + 9y_1^2 = -20x_1 + 25$ then $10x_1^2 + 18y_1^2 = -40x_1 + 50$,

$\Rightarrow y = \dfrac{45x_1 + 180}{18y_1} = \dfrac{5(x_1+4)}{2y_1}$

b- F (-4;0)

$\overrightarrow{FN_1}(x_1+4; y_1)$ *and* $\overrightarrow{FN_2}(x_2+4; y_2)$

N_1 , N_2 and F are collinear so $\overrightarrow{FN_1}$ and $\overrightarrow{FN_2}$ are collinear hence,

$y_2(x_1+4) = y_1(x_2+4)$

$x_1y_2 - y_1x_2 = 4(y_1 - y_2)$

Let T_2 be the tangent to (E) at N_2, (T_2) cuts (δ) at point J such that $\dfrac{5(x_2+4)}{2y_2}$

$x_1y_2 - y_1x_2 = 4(y_1 - y_2)$ then $\dfrac{x_2+4}{y_2} = \dfrac{x_1+4}{y_1}$

$\dfrac{5}{2} \times \dfrac{(x_2+4)}{y_2} = \dfrac{5}{2} \times \dfrac{(x_1+4)}{y_1}$ *then* $y_J = y_1$ and since I and J have the same abscissa then I *and* J are confouned. Consequently (T_1) and (T_2) intersecton (δ)

c- $(N_1 N_2)$ is a focal secant passing through F. (T_1) and (T_2) intersect on (δ) at I by symmetry the secant (MM′) passes through O and the tangents to (E) at M and M′ intersect on the directrix (d).

IV- A-1) $2y' - y = 0$ is equivalent at $y' - \dfrac{1}{2}y = 0$ the general solution of (E) is $y = Ce^{\frac{x}{2}}$

2) a- The representative curve of g passes through the point $(0;\sqrt{e})$ then

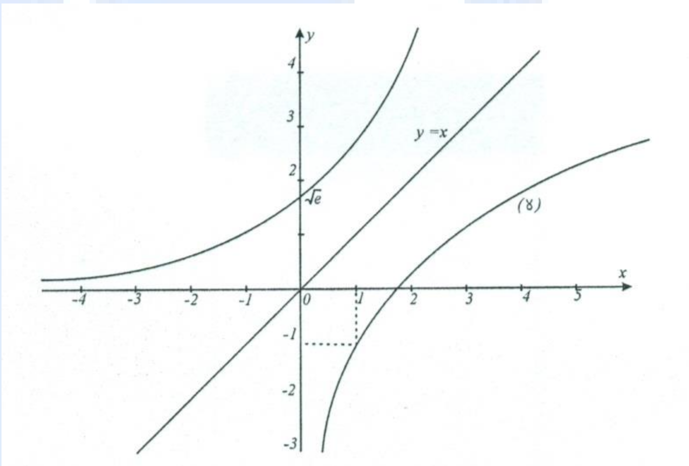
$\sqrt{e} = Ce^0$ *then* $C = \sqrt{e}$ $\qquad \Rightarrow g(x) = \sqrt{e}\ e^{\frac{x}{2}} = e^{\frac{x+1}{2}}$

b- g is continue and strictly increasing over IR then it admits an inverse function g^{-1}

$D_{g^{-1}} =]0; +\infty[$

c- The curve (γ) of g^{-1} is the symmetric of the curve of g with respect to the first bisector

$y = e^{\frac{x+1}{2}} \Rightarrow x+1 = 2\ln y \Rightarrow x = 2\ln y - 1 \Rightarrow g^{-1}(x) = 2\ln x - 1$



B) a- * If $0 < x < 1$ then $0 < \frac{x}{e} < \frac{1}{e} < 1 \Rightarrow \ln x < 0$ and $\ln\frac{x}{e} < 0$

$$f(x) = |\ln x| + \left|\ln\frac{x}{e}\right| = -\ln x - \ln\frac{x}{e}$$

$$= -\ln x - \ln x + \ln e = 1 - 2\ln x$$

* If $1 \le x < e \Rightarrow \frac{1}{e} \le \frac{x}{e} \le 1$

$\ln x \ge 0$ and $\ln\frac{x}{e} \le 0$ therefore $f(x) = |\ln x| + \left|\ln\frac{x}{e}\right| = \ln x - \ln\frac{x}{e} = \ln x - \ln x + \ln e = 1$

* If $x > e$ then $\frac{x}{\mathrm{e}} > 1$, $\ln x > 0$ and $\ln\frac{x}{\mathrm{e}} > 0$

$f(x) = |\ln x| + \left|\ln \frac{x}{e}\right| = \ln x + \ln \frac{x}{e} = \ln x + \ln x - \ln e = 2\ln x - 1$

$$f(x) = \begin{cases} 1-2\ln x & \text{if } x \in]0;1[\\ 1 & \text{if } x \in [1;e] \\ 2\ln x - 1 & \text{if } x \in]e;+\infty[\end{cases}$$

b- $f(1) = 1$

$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} (1-2\ln x) = 1-2.0 = 1 = f(1)$

$\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+}(1) = f(1) \Rightarrow f$ *is continuous at* 1

- if $f(e) = 1$

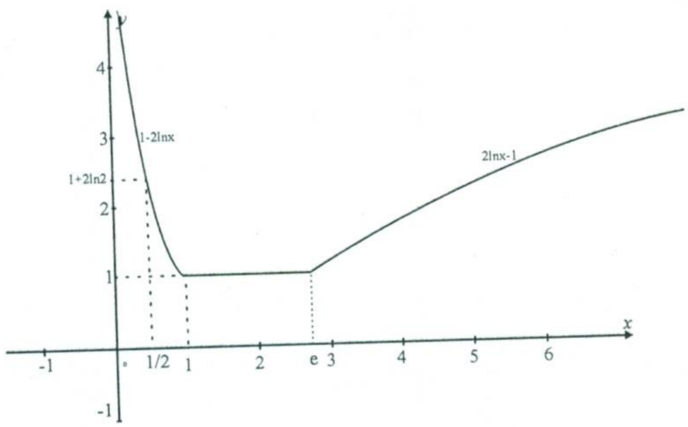
$\lim\limits_{x\to e^-} f(x) = \lim\limits_{x\to e^-}(1) = 1 = f(e) \Rightarrow f$ is continuous at e

$\lim\limits_{x\to e^+} f(x) = \lim\limits_{x\to e^+}(2\ln x - 1) = 1 = f(e)$

2) a- $f(\frac{1}{2}) = 1-2\ln \frac{1}{2} = 1+2\ln 2$

$f(e^2) = 2\ln e^2 - 1 = 4-1 = 3$

b-



3) $f(a) = |\ln a| + \left|\ln\frac{a}{e}\right| = |\ln a| + \left|\ln\frac{1}{b}\right| = |\ln a| + |\ln b|$

$f(b) = |\ln b| + \left|\ln\frac{b}{e}\right| = |\ln b| + \left|\ln\frac{1}{a}\right| = |\ln b| + |\ln a|$ then $f(a) = f(b)$

4) a- For $\lambda > 1$, $f(x) = \lambda > 1$ graphically the straight line of equation $y = \lambda$ cuts (C) in two points of abscissas x_1 and x_2 such that

$x_1 \in]0,1[$ and $x_2 \in]e;+\infty[$ then $1 - 2\ln x_1 = 2\ln x_2 - 1$

$\ln x_1 + \ln x_2 = 1$ so $\ln x_1 x_2 = 1$ and then $x_1 x_2 = e$

The equation $f(x) = \lambda$ has two roots x_1 and x_2 such that $x_1 x_2 = e$

b- $f(x) = 1+\ln 4$ gives $x = \frac{1}{2}$ *or* $x = 2e$

$f(x) = 3$ gives $x = e^2$ or $x = \frac{1}{e}$

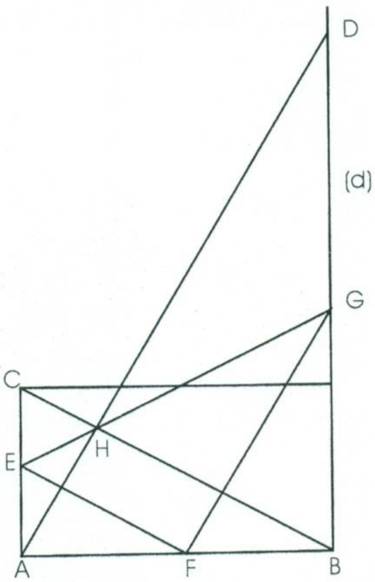
5) $y = e^{f(x)}$ gives $f(x) = \ln y$ and $f(x) = 2\ln x - 1$ since $x \geq e$ therefore $2\ln x - 1 = \ln y$, so $\ln x^2 - \ln e = \ln y$ or $\ln\frac{x^2}{e} = \ln y$ therefore $x^2 = ey$ then (P) is a part of parabola

whose focus axis is yy' of vertex O, focus F(0; $\frac{e}{4}$) and the directrix the straight line of equation y = -$\frac{e}{4}$

V) 1) h (C) = B then the image of (AC) is a straight lines passing through B and parallel to (AC) that is the straight line (d)

D = h(A) then D, H and A are collinear so D $\in (AH)$

$A \in (AC)$ then h (A) $\in (d)$, Therefore D is the intersection of the two straight lines (d) and(AH)



2) a- S(A) = B and S(C) = A then the angle of S is $(\vec{AC}, \vec{BA}) = \frac{\pi}{2}(2\pi)$

b- The image of (AH) by S is the straight line passing through B and perpendicular to (AH), so it is the straight line (BC).

The image of (CH) by S is the straight line passing through A and perpendicular to (CH), so it is the straight line (AH).

c- H is the intersection of the two straight lines (AH) and (CH) so its image by S is the point of intersection of (BC) and (AH) then it is the point H therefore S(H) = H, hence H is the center of S.

3) a- The image by S of (AB) is the straight line passing by B and perpendicular at (AB), then this is the straight line (BD).
The image by S of (CB) is the straight line passing by A and perpendicular at (CB), then this is the straight line (AD).

b- B is the intersection of the two straight lines (AB) and (CB) then its image by S is the point of intersection of the two straight lines (BD) and (AD) hence it is the point D
$S \circ S\ (A) = S\ (S(A)) = S(B) = D = h\ (A)$

c- $S \circ S = S\left(H; k; \frac{\pi}{2}\right) \circ S\left(H; k; \frac{\pi}{2}\right) = S\left(H; k^2; \frac{\pi}{2} + \frac{\pi}{2}\right) = S\left(H; k^2; \pi\right) = h(H; -k^2)$

$S \circ S$ and h have the same center and $S \circ S\ (A) = h(A)$ then $S \circ S = h$

4) a- E is the midpoint of [AC] , and since similitude preserves midpoints then S(E) is the midpoint of segment [BA] image of [AC] by S, hence F is the midpoint of [BA].
F is the midpoint of [BA] then S(F) is the midpoint of segment [BD] image of [BA] by S, consequently G is the midpoint of [BD].

b- $E \xrightarrow{s} F \xrightarrow{s} G$, *then* $E \xrightarrow{h} G$
So $\overrightarrow{HG} = -k^2\overrightarrow{HE}$ then the points E, H and G are collinear.
S(EF) = (FG), then the two straight lines (EF) and (FG) are perpendicular and the triangle EFG is right.

5) a- $z_B = 6$: $z_C = 4i$
The complex form of S is $z' = az + b$
S(A) = B gives $z_B = az_A + b$ then $b = 6$
S(C) = A gives $z_A = az_C + b$ then $a = \frac{3}{2}i$ therefore $z' = \frac{3}{2}iz + 6$
The ratio of S is $k = |a| = \frac{3}{2}$ and the ratio of h is $k' = -k^2 = \frac{-9}{4}$ h is a negative homothetic

b- $z_H = \frac{b}{1-a} = \frac{6}{1-\frac{3}{2}i} = \frac{12}{13}(2+3i)$ then $H\left(\frac{24}{13}, \frac{36}{13}\right)$

S(B) = D so $z_D = \frac{3}{2}iz_B + 6 = \frac{3}{2}i \times 6 + 6 = 6 + 9i$ then D (6; 9)