Entrance exam 2005-2006 **Physics exam** **Duration: 2 hours**

I- [17 pts] The simple pendulum as a harmonic oscillator

The aim of this study is to find the condition to be satisfied so that a simple pendulum can be considered as a harmonic oscillator.

A simple pendulum (P) is formed of a small bob of mass m = 200 g, suspended from a massless string of length ℓ = 1 m. (P), shifted by an angle α_0 with respect to the equilibrium position, is released at $t_o = 0$ without initial velocity. At an instant t, (P) makes with the vertical an angle α and moves with a velocity $\vec{V}$. Take g = 10 m/s^2 and neglect all frictional forces.

A. Theoretical study

1. The lower position of the bob is taken as zero gravitational potential energy. Give, at the instant t, the mechanical energy M.E of the system ((P), Earth) in terms of m, g, ℓ, α and V.

2. Applying the conservation of M.E, show that: $(\dot{\alpha})^2 = \left(\frac{d\alpha}{dt}\right)^2 = \frac{2g}{\ell}(\cos\alpha - \cos\alpha_0)$.

3. Show that the differential equation that describes the motion of (P) is given by: $\ddot{\alpha} + \frac{g}{\ell}\sin\alpha = 0$.

4. a) What is the approximation that must be taken in order to consider (P) as a harmonic oscillator?
   b) Deduce then the expression of the proper period T_0 of this harmonic pendulum and calculate its value.

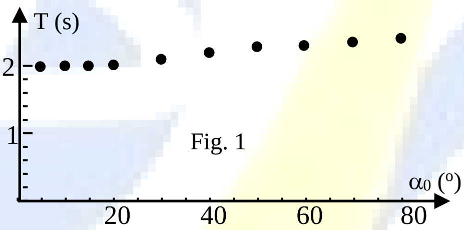


Fig. 1

B. Experimental study

With an appropriate device, we record three curves 1, 2 and 3. The curve 1 gives the variation of the period of the pendulum in terms of α_0 and the curves 2 and 3 give the variation of the angle α in terms of time for two different values of α_0.

1. Determine, from curve 1, the condition for which the pendulum behaves as a harmonic oscillator.
2. Determine, from curves 2 and 3, the periods T_1 and T_2 and the amplitudes α_{01} and α_{02} of oscillations of the pendulum in the two cases.
3. By comparing T_1 and T_2 with T_0, deduce that this pendulum does not always behave as a harmonic oscillator.
4. The condition in part A is in agreement with that of part B. Why?

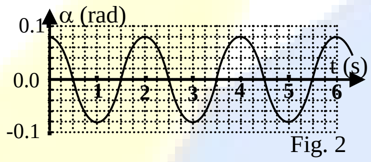


Fig. 2

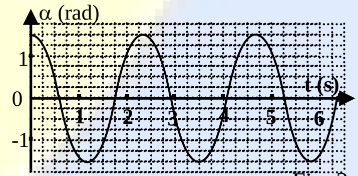


Fig. 3

II- [19 pts] Nuclear and tobacco

A The radioisotope polonium

The polonium ${}^{210}_{84}\text{Po}$ is a radioactive isotope of the element polonium which emits an α particle during its disintegration. The daughter nucleus is a stable isotope of lead (Pb).

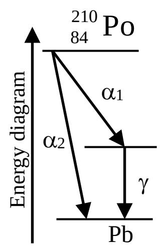


Fig. 4

1. Write, justifying it, the equation of the α disintegration of ${}^{210}_{84}\text{Po}$.
2. Calculate, in MeV and in J, the energy liberated by this disintegration.
3. Supposing that the polonium nucleus is initially at rest and that there is a conservation of the linear momentum:
   a) determine the speed of the lead daughter and the maximum speed of the α particle;
   b) deduce, in J and in MeV, the maximum kinetic energy of the α particle.
4. We notice that this disintegration is generally accompanied by the emission of a γ ray of wavelength $\lambda = 1.53\times10^{-12}$ m. Show that polonium 210 emits also α particles with a kinetic energy of about 4.5 MeV.

Given: 1 u = 1.66×0^{-27} kg = 931.5 MeV/c^2; e = 1.60×10^{-19} C; h= 6.63×0^{-34} J×s; c = 3 $\times10^8$ m/s;
m_{Po} = 209.9829 u; m_{Pb} = 205.9745 u; m_α = 4.0026 u.

B- Radioactive polonium in tobacco

For over 35 years, researchers and tobacco corporations have known that tobacco plants absorb lead 210 and polonium 210 from the soils in which they grow. In addition, tobacco plants gather naturally present radon 222 (${}^{222}_{86}\text{Rn}$) from the surrounding air. (*WWW.webspawner.com*)

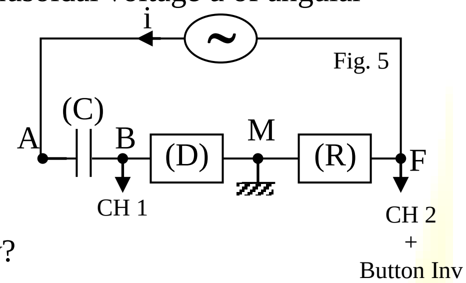
1. Radon 222 (${}^{222}_{86}\text{Rn}$), a natural gas, disintegrates through many α and β^- disintegrations and reaches finally the polonium 210. Determine the numbers x and y of α and β^- emissions respectively.
2. Lead 210, when it is disintegrating, emits one kind of radiation and reaches also the polonium 210. Determine, justifying it, the nature of this radiation?
3. When you light up a cigarette, the polonium is volatilized, you inhale it, and it is quickly deposited in the living tissues of the respiratory system. It is estimated that the energy absorbed from polonium 210 α radiations is about 1.2 J per year of smoking two packs of cigarettes per day.
   a) Which of the three radiations alpha, beta or gamma is the least penetrating?
   b) Deduce that α particle is more harmful for the lung than the other radiations.
4. If one chest x-ray could deliver in average 5×10^{-4} J to the same tissue, determine the equivalent number of chest x-rays that receives the lung in one year.
5. When you smoke a cigarette, you run a risk. What could this risk be?

III- [24 pts] Characteristics of a coil

In order to determine the characteristics of a coil (D), we use the circuit as shown in the figure 5 where (C) is a capacitor of capacitance C = 4 μF, (R) a resistor of resistance R = 200 Ω and the coil (L, r). (D), (R) and (C) are put in series with a generator presenting across its terminals an alternating sinusoidal voltage u of angular frequency ω.

In steady state, the circuit carries an alternating sinusoidal current i.

An oscilloscope, connected properly, allows to display the voltages $u_{BM} = u_D$ and $u_{MF} = u_R$ respectively across (D) and (R).



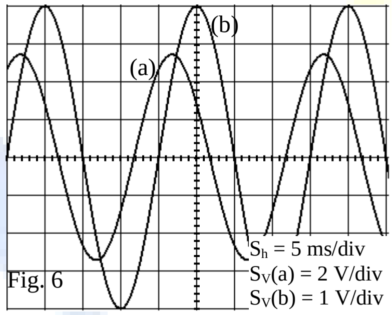


Fig. 6

A- Determination of the characteristics of the coil

1. In order to display u_{MF}, we had to push the button "Inv" of channel 2. Why?
2. Determine, using the oscillogram, the values of the period T of each curve and of ω.
3. a) Calculate the phase difference φ of the curve (a) with respect to the curve (b).
   b) Why can't we neglect the value of r?
4. The current i and the voltage u_D may be written respectively in the form: $i = I_m \sin\omega t$ and $u_D = (U_D)_m \sin(\omega t + \varphi)$.
   a) Determine I_m and $(U_D)_m$.
   b) Determine the expression of u_D in terms of t, r and L.
   c) Using two different values of "ωt", determine the values of L and r.

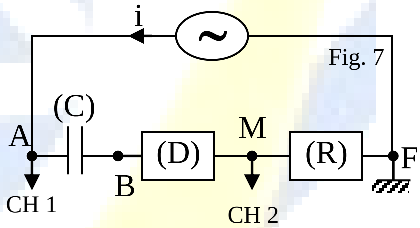
B- Verification of the value of r

1. Give two expressions of the average power consumed by (D).
2. Deduce that the value of r is in agreement with that obtained in A.

C- Verification of the value of L

We change the connections of the oscilloscope as shown in the figure 7.

We adjust the frequency of the voltage u so that the two curves are as shown in figure 8.



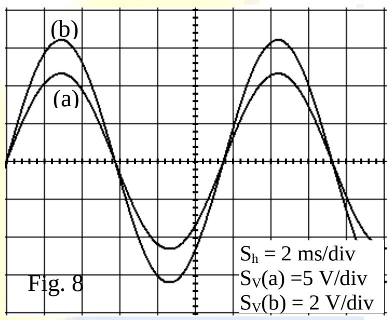


Fig. 8

1. What is the name of the observed phenomenon? Why?
2. a) Which of the two curves give the variation of the voltage u_{AF}? Why?
   b) Determine, using the curves, the period T_0 of the voltage u.
3. Deduce that the value of L is in agreement with that obtained in A.

Entrance exam 2005-2006 **Solution of Physics** **Duration: 2 hours**

I- A. Theoretical study [17 pts]

1. ME = 1/2 mV^2 + mgh; h = ℓ - $\ell\cos\alpha \Rightarrow$ ME = 1/2 $mV^2 + mg\ell(1-\cos\alpha)$

2. ME = constant = ME(t = 0) = 0 + $mg\ell(1 - \cos\alpha_0)$

1/2 $mV^2 + mg\ell(1-\cos\alpha) = mg\ell(1 - \cos\alpha_0) \Rightarrow V^2 = 2g\ell(\cos\alpha - \cos\alpha_0)$

However, in algebraic measurement, $V = \ell\dot{\alpha} = \ell\dfrac{d\alpha}{dt}$: $\left(\dfrac{d\alpha}{dt}\right)^2 = (\dot{\alpha})^2 = \dfrac{2g}{\ell}(\cos\alpha - \cos\alpha_0)$

3. Let us derive with respect to time: $2\dot{\alpha}\ddot{\alpha} = 2\dfrac{g}{\ell}(-\sin\alpha\,\dot{\alpha})$; however $\dot{\alpha} \neq 0$ during oscillations: $\Rightarrow$

$$\ddot{\alpha} + \frac{g}{\ell}\sin\alpha = 0$$

4. a) For an harmonic oscillator, the differential equation is of the form: $\ddot{\alpha} + \omega_0^2 \cdot \alpha = 0$;

b) However this is only possible if that $\alpha \leq 15^o$; because for $\alpha \leq 15^o$ $\sin\alpha \approx \alpha(rd) \Rightarrow \ddot{\alpha} + \dfrac{g}{\ell}\alpha = 0$.

$$\omega_0^2 = \frac{g}{\ell}\ ;\ \omega_0 = \sqrt{\frac{g}{\ell}} = \frac{2\pi}{T_0} \Rightarrow T_o = 2\text{ s}$$

B. Experimental study

1. According to the curve 1: T = 2 s for $\alpha < 20^o$. $T = T_0 = 2$ s.

2. $T_1 = 2$ s and $\alpha_{01} = 0.08$ rd ($\alpha_{01} < 0.1$ rd however 0.1 rd = $6^o < 15^o$)

- $T_2 \approx 2.33$ s et $\alpha_{02} = 1.5$ rd

3. $T_1 = T_0$ and $T_2 > T_0$.

This pendulum behaves like a harmonic oscillator when α is weak, that is to say $\alpha \leq 15^o$ (or experimentally $\alpha < 20^o$) ($\alpha_{02} \approx 86^o$)

4. Because $T_1 = T_0$ for $\alpha \leq 15^o$.

II- [19 pts] Nuclear and tobacco

A- The radioisotope polonium

1. ${}^{210}_{84}Po \rightarrow {}^{206}_{82}Pb^* + {}^{4}_{2}He$ Z et A

2. E = Δmc^2 = (209.9829 – 205.9745 – 4.0026)×931.5 = 5.4027 MeV or 8.644×10^{-13} J.

3. a) conservation of the linear momentum: $\vec{P}_{Pb} + \vec{P}_\alpha = \vec{0} \Rightarrow m_{Pb}V_1 = m_\alpha V_2$

Conservation of the total energy of the system:

$$E = E_{c(Pb)} + E_{cmax(\alpha)} = 1/2\, m_{Pb}V_1^2 + 1/2\, m_\alpha V_2^2 = 1/2\, m_{Pb}V_1^2 + \frac{1}{2m_\alpha}m_{Pb}^2 m_\alpha V_1^2$$

$$V_1 = \sqrt{\frac{2m_\alpha E}{m_{Pb}(m_\alpha + m_{Pb})}} \Rightarrow V_1 = 3.1\times10^5 \text{ m/s and } V_2 = \frac{m_{Pb}\cdot V_1}{m_\alpha} = 1.6\times10^7 \text{ m/s.}$$

b) $E_{cmax(\alpha)} = 1/2\, m_\alpha V_2^2 = 8.48\times10^{-13}$ J = 5.29 MeV

4. The energy of the emitted photon is given by:

$E_\gamma = \dfrac{hc}{\lambda} = \dfrac{6.63\times10^{-34}\cdot3\times10^{8}}{1.53\times10^{-12}} = 1.3\times10^{-13}$ J or 0.81 MeV

But $E_{cmax(\alpha)} = E_\gamma + E_{c(\alpha)} \Rightarrow E_{c(\alpha)} = 5.29 - 0.81 = 4.48$ MeV.

B- Radioactive polonium in tobacco

1. ${}^{222}_{86}Rn \rightarrow {}^{210}_{84}Po + x\,{}^{4}_{2}He + y\,{}^{0}_{-1}e$; $222 = 210 + 0 + 4x \Rightarrow x = 3$; $86 = 84 - y + 2x \Rightarrow y = 4$

2. Plomb 210 emits β^- because A remains the same and Z increases from 82 to 84

3. a) α is the least penetrating, because its mass is much larger than that of β.

b) Particles α remain then trapped in the lungs and thus give all their energy to the lungs.

4. $N = 1.2 / (5\times10^{-4}) = 2,4\times10^{3}$

The cigarette is dangerous, because it can cause the lung cancer to people.

III- [24 pts] Characteristics of a coil

A- Determination of the characteristics of the coil

1) The oscilloscope displays the voltage between a point and the ground. It can display u_{BM} and u_{FM}. To display u_{MF}, one must push the button Inv.

2) T covers 4 div, with s_h = 5 ms/div, on the two channels. We obtain:

T = 5 ms/div × 4 div = 20 ms, thus $T = 2\times10^{-2}$ s. The pulsation $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{2\cdot10^{-2}} = 100\,\pi$ or 314 rd/s

3. a) For the difference of phase: $\varphi = \dfrac{2\pi\times0.6\,\text{div}}{4\,\text{div}} = 0.3\pi$ or 0.94 rd

(a) leads (b) since (a) takes zero value before (b).

b) if R is negligible, we will have $u_D = L\dfrac{di}{dt}$ and u_D will lead i with a phase of $\pi/2$, which is not the case.

4- a) The maximum voltage across R is: RI_m = 1 V/div×4 div = 4 V $\Rightarrow I_m = U_m/R$; $I_m = \dfrac{4}{200} = 0.02$ A

$(U_D)_m$ = 2 V/div×2.8 div = 5.6 V.

b) $u_D = = r\,i + L\dfrac{di}{dt} = rI_m\sin(\omega t) + L\omega I_m\cos(\omega t)$

c) $u_D = (U_D)_m\sin(\omega t + \varphi) = rI_m\sin(\omega t) + L\omega I_m\cos(\omega t)$

For $\omega t = 0$: $(U_D)_m\sin(\varphi) = L\omega I_m \Rightarrow L = \dfrac{(U_D)_m\cdot c\sin\varphi}{I_m\cdot\omega} \approx 0.72$ H

For $\omega t = \pi/2$: $(U_D)_m\cos(\varphi) = rI_m \Rightarrow r = \dfrac{(U_D)_m\cdot\cos\varphi}{I_m} = 164.58\ \Omega \approx 165\ \Omega$.

B- Verification of the value of r

1. Average power: $P_{av} = rI^2 = U_D I \cos(\varphi)$.

2. a) $r = \dfrac{U_D \cdot \cos\varphi}{I} = \dfrac{(U_D)_m \cdot \cos\varphi}{I_m} = 165\ \Omega$

C- Verification of the value of L

1. It is the phenomenon of resonance of current because the voltage across the generator is in phase with the current which is displayed by the voltage across R.

2. a) For the curve (a): $(U_{(a)})_m = 2.3\times5 = 11.5$ V

For the curve (b): $(U_{(b)})_m = 3.2\times2 = 6.4$ V $\Rightarrow$ (a) represents u_{AF} the voltage across the generatort (b) represents u_{MF} the voltage across R.

Indeed, the maximum or effective values are always such as: $(U_G)_m >$: $(U_R)_m$ because the coil has a resistance. At resonance $(U_G)_m = (R+r)I_m$ and $(U_R)_m = RI_m$.

b) The proper period of the circuit: $T_0 = 2$ ms/div$\times5.75$ div $= 2\times10^{-3}\times5.75 = 11.5\times10^{-3}$ s.

3. At current resonance: $T_0 = 2\pi\sqrt{LC} \Rightarrow L = \dfrac{T_0^2}{4\pi^2 C}$ et L = 0.71 H.