Entrance Exam (2025-2026)
MATHEMATICS
TEST A
(Lebanese Program)

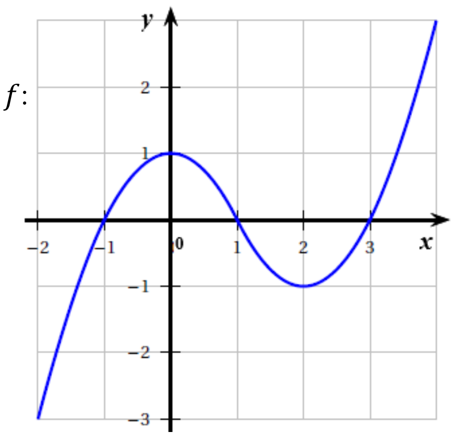
Date : 17/07/2025
Time : 2h30

Smartphones, notes and graphical calculators are strictly prohibited.

Exercise 1: (12 points)

This exercise is a multiple-choice quiz. For each question, ***only one*** *of the four answers given is correct. A wrong answer, a multiple answer or the absence of an answer to a question does not earn or deduct points. For this exercise, you have to answer on the answers sheet, by circling for each question* ***only one*** *of the letters* ***a***, ***b***, ***c*** *or* ***d***.
No justification is required.

1. The right figure shows the graph of the derivative f' of a function f, defined and differentiable on the interval $[-2\,;\,4]$.
Using a graphical reading of the curve of f', determine the correct statement about f:



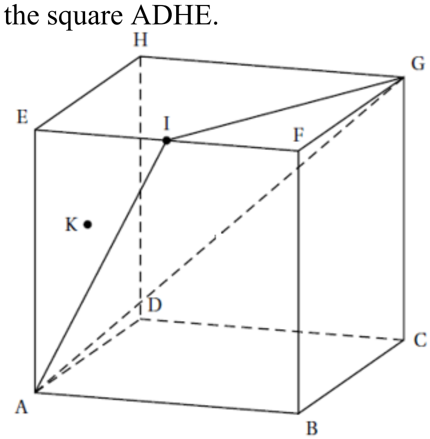
a) f is decreasing on $[0\,;2]$.
b) f is decreasing on $[-1\,;0]$.
c) f has a maximum at 1 on $[0\,;2]$.
d) f has a maximum at 3 on $[2\,;4]$.

2. Consider the sequence (U_n) defined by $U_0 = 1$ and $U_{n+1} = \frac{U_n}{3-U_n}$ for all $n \in \mathbb{N}$.
The sequence (V_n) defined by $V_n = \frac{1}{U_n} - \frac{1}{2}$ is:

a) arithmetic. **b)** geometric. **c)** constant. **d)** None of the preceding statements is true.

3. We consider the cube ABCDEFGH with an edge length of **1**. The space is referred to the orthonormal coordinate system $(A\,;\overrightarrow{AB},\overrightarrow{AD},\overrightarrow{AE})$. Point I is the midpoint of the segment [EF], and K is the center of the square ADHE.

a) The angle $\widehat{AIG}$ is equal to 90° .
b) The vector $\overrightarrow{BK}$ is normal to the plane (AIG).
c) The line (BK) intersects the plane (AIG) at the point $L(\frac{1}{3};\ 0;\ 1)$.
d) None of the preceding statements is true.



4. An exponential form of the complex number $z = \frac{(\sqrt{3}-i)\left(\cos\left(\frac{\pi}{5}\right)+i.\sin\left(\frac{\pi}{5}\right)\right)^3}{(1+i)^2}$ is:

a) $z = \frac{1}{2}e^{-i\frac{\pi}{15}}$ **b)** $z = e^{-i\frac{\pi}{15}}$ **c)** $z = e^{i\frac{\pi}{15}}$ **d)** None of the preceding answers is correct.

5. If $3x - y = 16$ then $\sqrt{\frac{8^{x-1}}{2^{y+1}}} =$

a) 3^3 **b)** 2^6 **c)** 2^{10} **d)** None of the preceding answers is correct.

6. Let f be the function given by $f(x) = \begin{cases} \frac{ax^2+(a+1)x+1}{2x^2+x-1} & \text{if } x \neq -1 \\ \frac{1}{3} & \text{if } x = -1 \end{cases}$ where a is a constant real number.

a) The function f is continuous at -1 if $a = -1$.
b) The function f is continuous at -1 if $a = \frac{1}{3}$.
c) The function f is continuous at -1 if $a = 2$.
d) None of the preceding statements is true.

7. Consider the function defined by $f(x) = x^{1000} + x$.

a) The function f is concave.
b) The graph of f has an inflection point of abscissa 0.
c) The graph of f has no inflection point.
d) None of the preceding statements is true.

8. Let $z \in \mathbb{C}$ such that $|3z + i| = |3iz + 1|$. Then we deduce that:

a) z is necessarily purely imaginary.
b) z is necessarily real.
c) z is necessarily zero.
d) None of the preceding statements is true.

9. Let f be the function defined on $\mathbb{R}_+^*$ by $f(x) = 2\left(\frac{1+\ln(x)}{x}\right)$.
In a company, the monthly profit (which may be negative), in thousands of euros, has been modeled by the function f on the interval $[0.2\ ;\ +\infty[$, based on the sale of x thousand manufactured items. Then:

a) The company's monthly profit can exceed 2100 euros.
b) The company's maximum monthly profit is 2000 euros.
c) The company will incur a loss if it sells more than 1000 units.
d) None of the above statements is true.

10. Consider the equation $3\ln(x+1) - 2\ln(x) = \ln(x+7)$, of real unknown x.
The solution set S of this equation is:

a) $S = \{\frac{1}{4}\ ;\ -1\}$
b) $S = \{-\frac{1}{4}\ ; 1\}$
c) $S = \{\frac{1}{4}\ ; 1\}$
d) $S = \{1\}$

11. An industrial machine loses 5% of its value each year due to wear and tear. The initial value of the machine is 10000 euros. After how many years will the machine have lost 50% of its value?

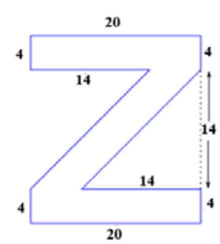
a) 10 years.
b) 7 years.
c) 14 years.
d) None of the preceding answers is correct.

12. The solution f of the differential equation $y' + 2y = -2$ satisfying $f(1) = 0$ is the function defined on $\mathbb{R}$ by:

a) $f(x) = e^{-2x} - e^{-2}$
b) $f(x) = -x^2 - 2x + 3$
c) $f(x) = e^{2-2x} - 1$
d) None of the preceding answers.

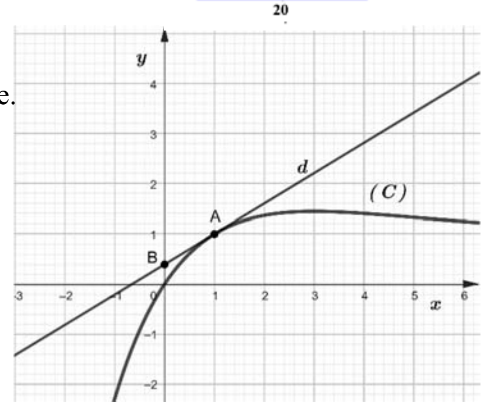
13. What is the surface area of the following Z-shaped figure?

a) 168
b) 220
c) 240
d) 244



14. Let f be the function defined on $\mathbb{R}$ by $f(x) = (x-1)e^{-kx} + 1$ where k is a constant real number.
The graph *(C)* of f is given on the right figure, in a system $(O, \vec{\imath}, \vec{\jmath})$ of the plane.
The tangent line d to the graph *(C)* at the point A of abscissa 1 passes through point $B(0\ ; 1 - \frac{1}{\sqrt{e}})$. Hence:

a) $k = 1$.
b) $k = -1$.
c) $k = \frac{1}{2}$.
d) None of the preceding statements is true.



15. The solution set of the inequality $2\cos(x) - \sqrt{3} \geq 0$ on the interval $[0\ ; 2\pi]$ is:

a) $[\frac{\pi}{6}\ ;\ 2\pi]$
b) $[-\frac{\pi}{6}\ ;\ \frac{\pi}{6}]$
c) $\left[0\ ; \frac{\pi}{6}\right] \cup [\frac{11\pi}{6}\ ; 2\pi]$
d) None of the preceding answers is correct.

16. To keep its fleet in good working condition, a very large transport company decides to have its vehicles inspected.
It is known that 20% of the vehicles are under warranty. Among the vehicles under warranty, 1% are defective, and among those no longer under warranty, 10% are defective.
A vehicle is randomly selected from the fleet. The probability that this vehicle is defective is:

a) 11 % **b)** 8.2 % **c)** 0.082 % **d)** None of the preceding answers is correct.

17. The limit, as n tends to $+\infty$, of the numerical sequence (U_n) defined by $U_n = n(e^{\frac{1}{n}} - 1)$ is equal to:

a) 1 **b)** $-\infty$ **c)** $+\infty$ **d)** 0

18. Two fair dice are rolled. What is the probability that the sum of the numbers obtained is greater than or equal to 11?

a) $\frac{1}{36}$ **b)** $\frac{1}{12}$ **c)** $\frac{1}{6}$ **d)** None of the preceding answers is correct.

19. Consider the following differential equation (E): $y' + y = 2\sin(x)$.
Let f be the unique solution of (E) such that $f(0) = 2$. An equation of the tangent to the graph of f at the point of abscissa 0 is :

a) $x = y - 2$ **b)** $y = -2x + 2$ **c)** $y = 2$ **d)** None of the preceding answers is correct.

20. (U_n) is a geometric sequence such that $U_0 = 2$ and $U_3 = \frac{128}{125}$.
Hence the sequence (S_n) given by $S_n = U_0 + U_1 + \cdots + U_n$ tends to:

a) 0 **b)** 10 **c)** $+\infty$ **d)** None of the preceding answers is correct.

21. $\lim\limits_{x\to 0^+} \dfrac{\ln^2(1+\sqrt{x}) + e^{2x} - 1}{x} =$

a) $+\infty$ **b)** 0 **c)** 3 **d)** None of the preceding answers is correct.

22. The complex plane is referred to a direct orthonormal system $(O, \vec{u}, \vec{v})$. Each point M of affix $z \neq -i,$ is associated with the point M' of affix z' such that $z' = \frac{z-i}{z+i}$.
If point M moves along the real axis then point M' moves along:

a) the real axis. **b)** the imaginary axis. **c)** the trigonometric circle. **d)** None of the preceding statements is true.

23. Below is the graph of a function f defined and continuous on the interval $I = [-3\,;\,8]$.
We define the function F on I by $F(x) = \int_0^x f(t)\,dt$.

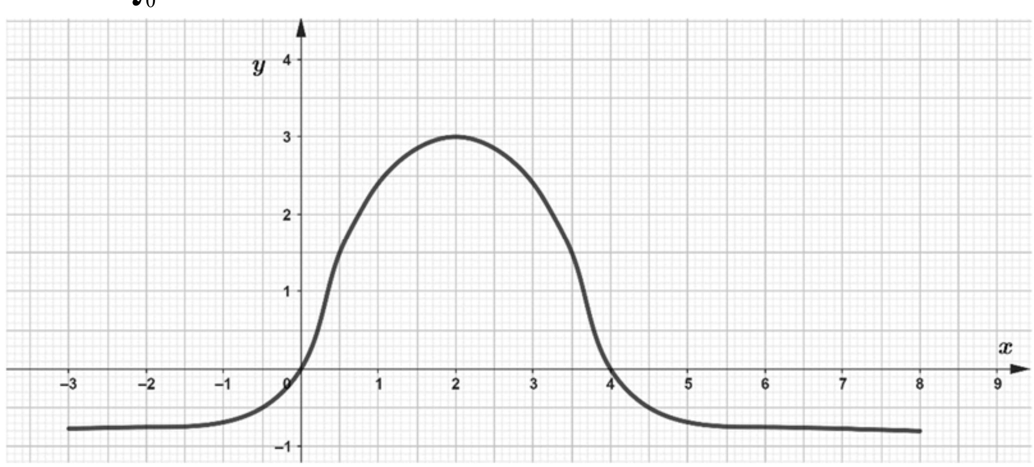
Hence :

a) $0 \le F(4) \le 4$

b) $6 \le F(4) \le 12$

c) $-1 \le F(4) \le 4$

d) None of the preceding statements is true.



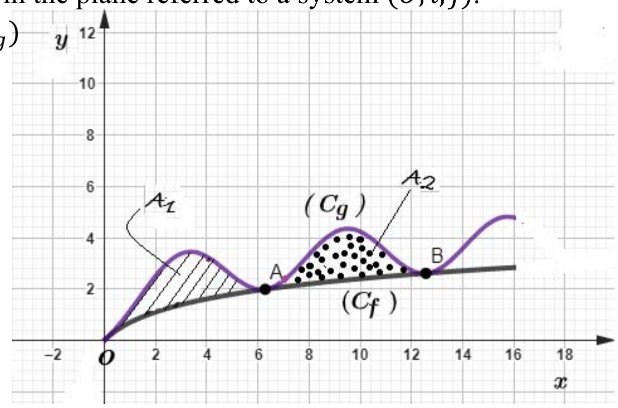
24. The algebraic form of the complex number $z = \frac{(1+i)^{13}}{(1-i)^{11}}$ is given by:

a) $z = 2 + i$ **b)** $z = 2i$ **c)** $z = -2$ **d)** None of the preceding answers is correct.

25. A factory produces mechanical items. It is estimated that 2% of the items produced by this factory are non-compliant. After inspection, it was found that 97% of the compliant items are accepted, and 99% of the non-compliant items are rejected. What is the probability of having an item which is compliant and rejected?

a) 0.03 **b)** 0.294 **c)** 0.0294 **d)** None of the preceding answers is correct.

26. Let f and g be the functions defined on the interval $[0; 16]$ by
$f(x) = \ln(x+1)$ and $g(x) = \ln(x+1) + 1 - \cos(x)$.
(C_f) and (C_g) are respectively the graphs of the functions f and g in the plane referred to a system $(O, \vec{i}, \vec{j})$.
A and B are the points of intersection of the two curves (C_f) and (C_g) on the interval $]0; 16]$. Let $\mathcal{A}_1$ be the shaded area between (C_g) and (C_f) on $[0; x_A]$, and let $\mathcal{A}_2$ be the dotted area between (C_g) and (C_f) on $[x_A; x_B]$.



We have:

a) $\mathcal{A}_1 \neq \mathcal{A}_2$

b) $\mathcal{A}_1 = \mathcal{A}_2 = (2\pi + 1)\ln(2\pi + 1)$

c) $\mathcal{A}_1 = \mathcal{A}_2 = 6$

d) None of the above statements is true.

27. In space, referred to an orthonormal system $(O, \vec{i}, \vec{j}, \vec{k})$, consider the planes (P) and (Q) with the following cartesian equations (P): $2x + y - z + 1 = 0$ and (Q): $x + 2y + z + 2 = 0$.

a) The planes (P) and (Q) are parallel.

b) The planes (P) and (Q) are perpendicular.

c) The planes (P) and (Q) intersect along a straight line (d) directed by the vector $\vec{u}(1\,;-1\,;1)$.

d) None of the above statements is true.

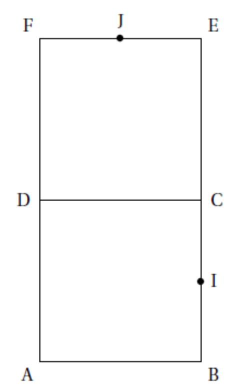
28. An urn contains three red balls, three black balls, and four white balls. Three balls are drawn successively without replacement**.** What is the probability that all the drawn balls are of the same color?

a) 0.2 **b)** 0.05 **c)** 0.118 **d)** None of the preceding answers is correct.

29. A farmer has 100 kg of watermelons. At the beginning, they are composed of 99% water and therefore 1% dry matter. Later, during storage, their water content decreases to 98%. What is the total weight of the watermelons at that point?

a) 98 kg **b)** 99 kg **c)** 50 kg **d)** None of the preceding answers is correct.

30. We consider two direct squares ABCD and DCEF, each with side length 1.
Point I is the midpoint of [BC] and point J is the midpoint of [EF] (see figure).
Let s be the direct plane similitude that maps point A to point I and point C to point J.
The plane is referred to the system $(A, \overrightarrow{AB}, \overrightarrow{AD})$.Then the complex representation of s is:



a) $z' = \left(1 + \frac{1}{2}i\right)z + \frac{1}{2} + i$

b) $z' = \left(\frac{1}{2} + i\right)z + 1 + \frac{1}{2}i$

c) $z' = \left(\frac{1}{2} - i\right)z + 1 + \frac{1}{2}i$

d) None of the above statements is true.

Exercise 2: (8 points)

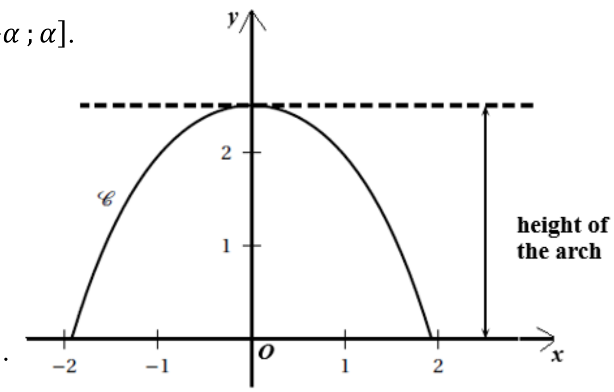
Part A:

Let f be the function defined on the set $\mathbb{R}$ of real numbers by: $f(x)=\frac{7}{2}-\frac{1}{2}(e^{x}+e^{-x})$.

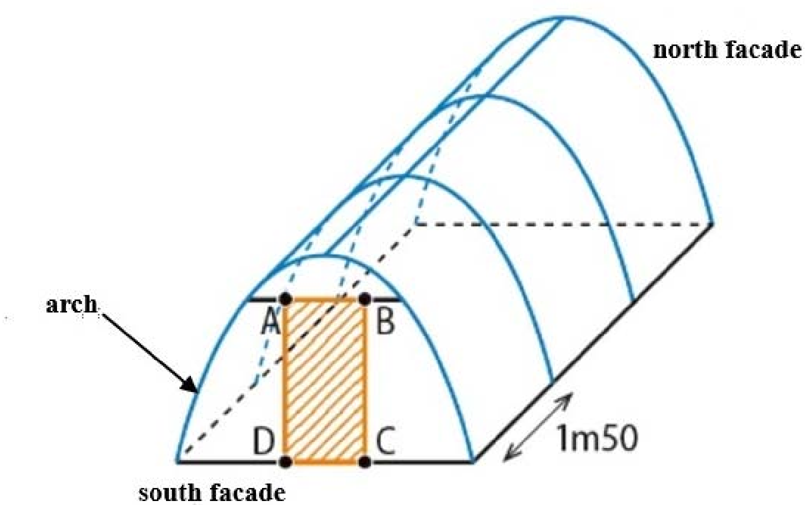
1. Determine the limit of the function f at $+\infty$.
2. Show that the function f is strictly decreasing on the interval $[0\,;+\infty[$.
3. Prove that the equation $f(x)=0$ has a unique solution α on the interval $[0\,;+\infty[$.
4. Justify that the equation $f(x)=0$ has exactly two solutions in $\mathbb{R}$, and that they are opposite.
5. Consider the numerical sequence (U_n) defined by $U_0=1$ and $U_{n+1}=\ln(7-e^{-U_n})$ for all integer $n\in\mathbb{N}$.
   - **a)** Using a proof by induction, show that for all integer $n\in\mathbb{N}$, we have: $0<U_n<U_{n+1}<2$.
   - **b)** Deduce that the sequence (U_n) is convergent. What is its limit?
6. Show that $1.92<\alpha<1.93$.

Part B :

Tunnel-shaped greenhouses are frequently used for growing delicate plants; they protect the plants from cold weather or large temperature fluctuations.
They are built using several identical metal arches that are fixed to the ground, and which support a plastic cover (sheeting).
In an orthonormal coordinate system with a unit of 1 meter, one arch of the greenhouse is modeled by the graph $\mathscr{C}$ of the function f, studied in Part A, over the interval $[-\alpha\,;\,\alpha]$.
In the figure below, we have represented the curve $\mathscr{C}$ on the interval $[-\alpha\,;\alpha]$.



1. Calculate the height of an arch (see figure).
2. In this question we aim to calculate the length of an arch, that is, the length of the curve $\mathscr{C}$.
   It is assumed that the length of the curve on the interval $[0\,;\alpha]$ is given, in meters, by the following integral: $I=\int_0^{\alpha}\sqrt{1+(f'(x))^2}\,dx$.
   - **a)** Prove that, for all real x, we have: $1+(f'(x))^2=\frac{1}{4}\left(e^{x}+e^{-x}\right)^2$.
   - **b)** Calculate I in terms of α. Deduce that the length L of an arch, in meters, is equal to $e^{\alpha}-e^{-\alpha}$.
3. We want to build a tunnel-shaped garden greenhouse.
   Four metal arches, shaped as described in the previous part, are fixed to the ground and spaced 1.5 meters apart, as shown in the figure below.



On the south facade, an opening is done, represented in the figure by the rectangle ABCD, with a width of 1 meter and a length of 2 meters.
We want to determine the amount of plastic sheeting, expressed in square meters, needed to build this greenhouse.
This plastic sheeting consists of three parts: one covering the north facade, another covering the south facade *(except for the opening)*, and a third rectangular part covering the roof of the greenhouse.

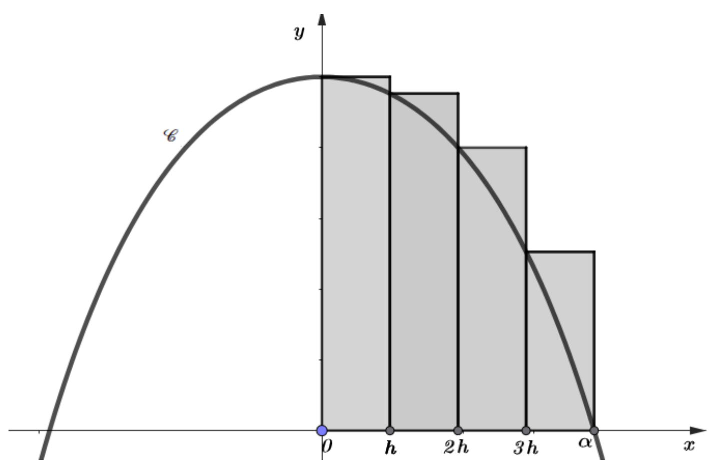
a) Show that the amount of plastic sheeting, in square meters, needed to cover the south and north facades is given by:
$$Q = 4\int_0^{\alpha} f(x)dx \; - 2$$

b) Use 1.925 as an approximate value for α throughout the rest of the problem.
Determine, to the nearest square meter, the total area $\mathcal{A}$ of plastic sheeting needed to build this greenhouse.

4. In this question, we aim to approximate the total area $\mathcal{A}$ of the plastic sheeting using a numerical method. First, we will compute an approximate value of the integral $S = \int_0^{\alpha} f(x)dx$ using the following numerical method (known as the ***rectangle method***):
We divide the interval $[0\,;\alpha]$ into n subintervals of equal length $h = \frac{\alpha}{n}$ with $n \in \mathbb{N}^*$. We then assume that an approximate value of the integral $S = \int_0^{\alpha} f(x)dx$ is given by the sum of the areas of the n rectangles, each with base h and with heights $f(kh)$ where k ranges from 0 to $n-1$.

Hence $S = \int_0^{\alpha} f(x)dx \approx S_n$ where $S_n = h[f(0) + f(h) + f(2h) + ... + f((n-1)h)] = h\sum_{k=0}^{n-1} f(kh)$.

In the figure below, to help visualize the situation, these rectangles are shown for $n = 4$. In fact, the larger the value of n, the closer the sum of the areas of the rectangles approximates the value of the integral $S = \int_0^{\alpha} f(x)dx$.



You are given below a table of values of f with $h = 1.925/500$ (meaning we have taken $n = 500$).

x	0	h	$2h$	$3h$	……..	$499h$	***Total***
$f(x)$	2.5	2.499993	2.499970	2.499933	……..	0.012377	***879.915036***

a) Based on the values in this table, deduce that $S_{500} = 3.387673$
What does this result represent?

b) Using this numerical result, find again the approximation of the total area $\mathcal{A}$ of the plastic sheeting needed to build the greenhouse.

Lebanese University
Faculty of Engineering

Date : *17/07/2025*

Entrance Exam (2025-2026)
MATHEMATICS
TEST A
CORRECTION
(Lebanese Program)

Exercise 1:

Question	Answer
1	c
2	b
3	b
4	b
5	b
6	c
7	c
8	b
9	b
10	d
11	c
12	c
13	d
14	c
15	c
16	b
17	a
18	b
19	b
20	b
21	c
22	c
23	b
24	d
25	c
26	d
27	c
28	b
29	c
30	b

Exercise 2 :

Part A :

1. We have $\begin{cases} \lim\limits_{x\to+\infty} e^x = +\infty \\ \lim\limits_{x\to+\infty} e^{-x} = 0 \end{cases} \Rightarrow \lim\limits_{x\to+\infty} e^x + e^{-x} = +\infty \Rightarrow \lim\limits_{x\to+\infty} -\frac{1}{2}(e^x + e^{-x}) = -\infty \Rightarrow \lim\limits_{x\to+\infty} \frac{7}{2} - \frac{1}{2}(e^x + e^{-x}) = -\infty$

∴ $\lim\limits_{x\to+\infty} f(x) = -\infty$.

2.
- $\forall x \in \mathbb{R}$, $f'(x) = -\frac{1}{2}(e^x - e^{-x})$
- For all $x > 0$, we have $x > -x$; hence $e^x > e^{-x}$; so $f'(x) < 0$.

∴ Therefore the function f is strictly decreasing on $[0\ ;\ +\infty[$.

3. On $[0\ ;\ +\infty[$: The function f is continuous and strictly decreasing. Moreover $\begin{cases} .\ f(0) = 5/2 \\ .\ \lim\limits_{x\to+\infty} f(x) = -\infty \end{cases}$.

But $0 \in]-\infty\ ;\ 5/2]$

∴ Therefore, according to Intermediate Value Theorem, we deduce that the equation $f(x) = 0$ has a unique solution α on the interval $[0\ ;\ +\infty[$.

4. We notice that for every real $x \in \mathbb{R}$, $f(-x) = f(x)$. Hence f is an even function.
But we have just seen that there exists a unique number $\alpha \in [0\ ;+\infty[$ such that $f(\alpha) = 0$.
And since $f(-\alpha) = f(\alpha)$, then $-\alpha \in]-\infty\ ;0]$ and satisfies $f(-\alpha) = 0$.
If there were another solution $\beta \neq -\alpha$ in $]-\infty\ ;\ 0[$, then $-\beta$ would be a second solution in $]0\ ;+\infty[$, which is not possible.

∴ The equation $f(x) = 0$ has exactly two opposite solutions in $\mathbb{R}$, that are α and $-\alpha$.

<u>*Another explanation*</u>**:** For all $x \in \mathbb{R}$, we observe that $f(-x) = f(x)$. Therefore, f is an even function. This means that the graph of f is symmetric with respect to the y-axis.
But, according to question 3, we know that the graph of f intersects the x-axis only once on the interval $[0\ ;+\infty[$, at the point with abscissa α .Thus, by symmetry, we can conclude that the graph of f intersects the x-axis exactly twice, at the points with abscissas α and $-\alpha$.

5. a) <u>Base step:</u>

For $n = 0$: We have $U_0 = 1$ and $U_1 = \ln(7 - e^{-U_0}) = \ln(7 - e^{-1}) = 1{,}8919\ldots \Rightarrow 0 < U_0 < U_1 < 2$.
Thus, the proposition is clearly true for $n = 0$. → *the base case is established*

<u>Inductive step:</u>

Assume the statement is true for some integer $n \geq 0$, ie $0 < U_n < U_{n+1} < 2$. (IH): *Inductive hypothesis*
Let's show that the statement also holds for $n + 1$, that is: $0 < U_{n+1} < U_{n+2} < 2$.
We have: (IH) $\Rightarrow -2 < -U_{n+1} < -U_n < 0$

$\Rightarrow e^{-2} < e^{-U_{n+1}} < e^{-U_n} < 1$

$\Rightarrow -1 < -e^{-U_n} < -e^{-U_{n+1}} < -e^{-2}$

$\Rightarrow 6 < 7 - e^{-U_n} < 7 - e^{-U_{n+1}} < 7 - e^{-2}$

$\Rightarrow 0 < \ln(6) < \ln(7 - e^{-U_n}) < \ln(7 - e^{-U_{n+1}}) < \ln(7 - e^{-2}) = 1{,}926\ldots < 2$

Therefore $0 < U_{n+1} < U_{n+2} < 2$. → *the inductive step holds*

∴ In conclusion, the proposition is true for all $n \in \mathbb{N}$ (by mathematical induction).

<u>*Another method*</u>*: By observing that $U_{n+1} = g(U_n)$ with $g(x) = \ln(7 - e^{-x})$, we can use the fact that the function g is strictly increasing on [0 ; 2] to prove the inductive step.*

b)
- According to the double inequality obtained in the preceding question, we deduce that the sequence (U_n) is increasing and is bounded above by 2.
Therefore, according to Monotone Convergence Theorem, the sequence is convergent.

- Let $L = \lim_{n\to+\infty} U_n \quad (0 < L \leq 2)$.

  Thus $L = \lim_{n\to+\infty} U_{n+1} = \lim_{n\to+\infty} \ln(7 - e^{-U_n}) = \ln(7 - e^{-L})$

  $\Rightarrow \; L = \ln(7 - e^{-L})$

  $\Rightarrow \; e^L = 7 - e^{-L}$

  $\Rightarrow \; 7 = e^L + e^{-L}$

  $\Rightarrow \; \frac{7}{2} = \frac{1}{2}(e^L + e^{-L})$

  $\Rightarrow \; f(L) = 0$

  $\Rightarrow \; L = \alpha \quad$ (car $\; L > 0$)

  $\therefore$ The sequence (U_n) converges to α.

6. We observe that: $\begin{cases} f(1{,}92) = 0{,}0162 \ldots > 0 \\ f(1{,}93) = -0{,}017 \ldots < 0 \end{cases}$. This implies that: $1.92 < \alpha < 1.93$.

Part B :

1. The height of an arch is: $f(0) = \frac{7}{2} - \frac{1}{2}(e^0 + e^0) = \frac{5}{2} = 2{,}5$ m.
2. **a)** $\forall x \in \mathbb{R}, \; 1 + \left(f'(x)\right)^2 = 1 + \frac{1}{4}(e^x - e^{-x})^2 = \frac{1}{4}[4 + (e^x - e^{-x})^2] = \frac{1}{4}[2 + e^{2x} + e^{-2x}] = \frac{1}{4}(e^x + e^{-x})^2$.

   b) • $I = \int_0^{\alpha} \sqrt{1 + (f'(x))^2}\, dx = \int_0^{\alpha} \frac{1}{2}(e^x + e^{-x}) dx = \frac{1}{2}\left[e^x - e^{-x}\right]_0^{\alpha} = \frac{1}{2}(e^{\alpha} - e^{-\alpha})$.

   • Since the function f is even, its graph is symmetric with respect to the y-axis, hence the length of the curve (meaning that of the arch) is : $L = 2I = e^{\alpha} - e^{-\alpha}$.

3. **a)** Each of the north and south façades has an area equal to $\int_{-\alpha}^{\alpha} f(x)dx = 2\int_0^{\alpha} f(x)dx$. The area of the opening is 2.

   Therefore, the amount Q of plastic sheet required to cover the south and north façades is given, in square meters, by:

   $Q =$ area of the south façade $+$ area of the north façade $-$ area of the opening

   $= 2\int_0^{\alpha} f(x)dx + 2\int_0^{\alpha} f(x)dx - 2 = 4\int_0^{\alpha} f(x)dx - 2$.

   b) The total area of the plastic sheet required to cover the entire greenhouse is given by:

   $\mathcal{A} = Q +$ area of the lateral sheet covering the roof.

   • $Q = 4\int_0^{\alpha} f(x)dx - 2 = 4\int_0^{\alpha} \left(\frac{7}{2} - \frac{1}{2}\left(e^x + e^{-x}\right)\right)dx - 2 = \int_0^{\alpha} \left(14 - 2\left(e^x + e^{-x}\right)\right)dx - 2 = 14\alpha - 2(e^{\alpha} - e^{-\alpha}) - 2$.

   • The area of the lateral roof sheet is that of a rectangle of length of 3 × 1.5 = 4.5 meters and of width of $L = 2I = e^{\alpha} - e^{-\alpha}$.

   Thus, this area is $4{,}5 \times \left(e^{\alpha} - e^{-\alpha}\right)$.

   $\therefore$ Therefore, the total area of the plastic sheet required to cover the entire greenhouse is then:

   $\mathcal{A} = Q + 4{,}5 \times \left(e^{\alpha} - e^{-\alpha}\right) = 14\alpha - 2(e^{\alpha} - e^{-\alpha}) - 2 + 4{,}5 \times \left(e^{\alpha} - e^{-\alpha}\right) = \boxed{14\alpha + 2{,}5(e^{\alpha} - e^{-\alpha}) - 2} \approx 41{,}72 \approx 42 \text{ m}^2$.

4. **a)** $S_{500} = h\sum_{k=0}^{500-1} f(kh) = \frac{1{,}925}{500} \times 879{,}915036 = 3{,}387673$.

   This value represents an approximation of the integral $S = \int_0^{\alpha} f(x)dx$.

   b) We have: $\mathcal{A} = Q + 4{,}5 \times \left(e^{\alpha} - e^{-\alpha}\right)$

   $= 4\int_0^{\alpha} f(x)dx - 2 + 4{,}5 \times \left(e^{\alpha} - e^{-\alpha}\right)$

   $= 4S - 2 + 4{,}5 \times (e^{\alpha} - e^{-\alpha})$

   $\approx 4 \times 3{,}387673 - 2 + 4{,}5(e^{1{,}925} - e^{-1{,}925}) \approx 41{,}74 \approx 42 \text{ m}^2$