Lebanese University **Entrance exam 2025-2026** **July 2025**

Faculty of Engineering **Physics (A)** **duration 120 min**

In order to eliminate random answer strategies, **each correct answer is rewarded by 1.0 point**, while **each wrong answer is penalized by the withdrawal of 0.5 point**.

Exercise 1: The Age of the Earth (21 pts)

The accurate determination of the Earth's age can be obtained by uranium-lead dating.
The naturally radioactive uranium 238 nucleus transforms, according to the radioactive family, into a stable lead 206 nucleus through a series of successive decays: ${}^{238}_{92}U \longrightarrow {}^{A}_{Z}Th \longrightarrow Pa \longrightarrow \ldots\ldots \longrightarrow {}^{206}_{82}Pb$.

Symbol	${}^{238}_{92}U$	${}^{4}_{2}He$	${}^{A}_{Z}Th$	proton	neutron
Mass (in u)	238.0003	4.0015	233.9942	1.0073	1.0087

c = 3×10^{8} m/s ; 1 u = 1.66×10^{-27} kg = 931.5 MeV/c^2 ; 1 eV = 1.60×10^{-19} J ; h = 6.63×10^{-34} J.s.
half-life of uranium 238 : $t_{½}$(${}^{238}_{92}U$) = 4.5×10^{9} years.

A- Decay of the Uranium 238 Nucleus

A uranium nucleus ${}^{238}_{92}U$ undergoes α radioactivity. The daughter nucleus is thorium ${}^{A}_{Z}Th$.

1. The gamma emission is not taken into account.

1.1. Write the reaction of this decay, specifying the value of Z and that of A.

1.2. Determine the mass lost Δm due to this decay.

1.3. Deduce that the energy released by this decay is 4.2849 MeV.

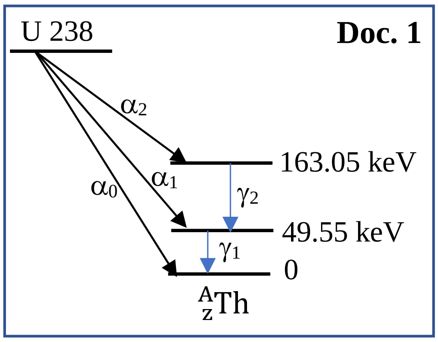
1.4. Assume that the uranium nucleus was at rest.

1.4.1. Determine the relationship between the kinetic energy KE(Th) of the daughter nucleus and KE(α) that of the α particle.

1.4.2. Deduce the value of KE(Th) and that of KE(α) knowing that KE(Th) = 0.0171×KE(α).

2. The gamma emission is taken into account.

During uranium 238 nuclei decay, the thorium nucleus, the daughter nucleus, can be in the ground state with energy E_0 = 0 or in one of the two excited states with energy E_1 = 0.04955 MeV or E_2 = 0.16305 MeV (Doc. 1). We will then have the emission of α particles forming three homokinetic groups α_0, α_1, and α_2. We assume that KE(Th) keeps the same value as calculated previously.



2.1. Knowing that: ${}^{238}_{92}U \rightarrow {}^{A}_{Z}Th^{*} + \alpha$, the thorium being in an excited state, determine the kinetic energies of the different α particles emitted.

2.2. Determine the wavelengths of the two radiations γ_1 and γ_2.

3. Nucleus stability

3.1. Determine the binding energy of the nucleus ${}^{238}_{92}U$.

3.2. The binding energy of ${}^{A}_{Z}Th$ nucleus is E_b = 1784.3814 MeV. Deduce the nucleus that is more stable than the other.

B- Geochronology

The naturally radioactive uranium 238 nucleus transforms, according to the radioactive family, into a stable lead 206 nucleus through a series of successive decays:

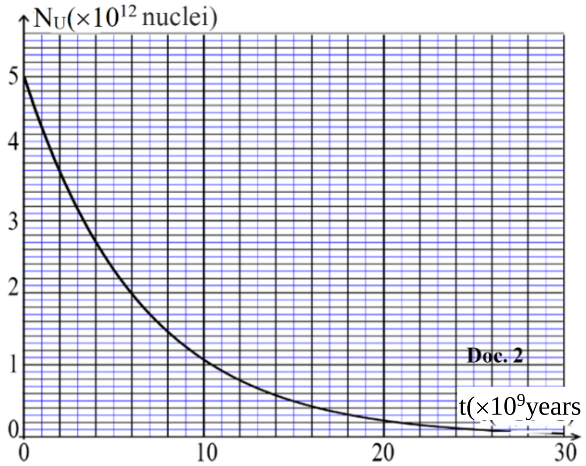
${}^{238}_{92}U \xrightarrow{\alpha} {}^{A}_{Z}Th \xrightarrow{\beta^-} Pa \longrightarrow \ldots\ldots \longrightarrow {}^{206}_{82}Pb$.

1. The overall equation for the transformation process of a uranium 238 nucleus into a lead 206 nucleus is:

${}^{238}_{92}U \rightarrow {}^{206}_{82}Pb + x({}^{0}_{-1}e) + y\,({}^{4}_{2}He)$.

Determine x and y using the specific laws.

2. Consider a sample of ancient rock formed at the instant t_0 = 0, the instant of formation of the Earth, which contains only uranium 238, with radioactive constant λ and whose number of nuclei is $N_U(0)$. At an instant t, this rock is found to contain uranium 238 and lead 206. The curve in (Doc. 2) shows the radioactive decay of the number $N_U(t)$ of uranium 238 nuclei in this rock, whose age, denoted t_{Earth}, corresponds to that of the Earth.



2.1. The number of uranium 238 nuclei, at an instant t, is expressed by $N_U(t) = B\cdot e^{\frac{-t}{\tau}}$. Determine the expressions for the constants B and τ in terms of the data.

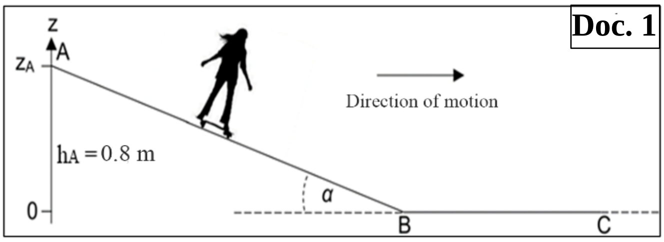
2.2., Referring to the graph (Doc. 2), determine the value of the radioactive constant λ of uranium 238.

2.3. The number of lead 206 nuclei present in the rock at the instant t_{Earth}, denoted $N_{pb}(t_{Earth})$, is equal to 2.5×10^{12}. Deduce, with justification, the number of uranium nuclei $N_U(t_{Earth})$ at this same instant.

2.4. Determine, then, the age t_{Earth} of the Earth.

Exercise 2: An Olympic Practice (14 pts)

We study several simple motion phases performed by a skateboarder. The set {Skateboard + skater}, denoted by (S) with mass m = 76 kg, is modeled by a point mass placed at its center of mass G (Doc. 1). Take the horizontal plane containing B and C as the reference level for the gravitational potential energy and $g = 9.8\ m\ s^{-2}$.



Doc. 1

A- Sliding on the inclined plane

(S), starting from rest at point A, $z_A = 0.80$ m, on an inclined plane of length AB = 2 m and making an angle α with the horizontal plane, reaches point B where the system [(S), Earth] has the mechanical energy ME(B) = 548.72 J. During this phase, we consider that the friction forces are modeled by a constant force denoted $\vec{F}$ opposite to motion.

1. The gravitational potential energy of the system [(S), Earth] at point A is:

a) GPE= 568.32 J ; **b)** GPE= 595.84 J; **c)** GPE= 602.5 J

2. The variation ΔME of the mechanical energy between A and B is :

a) ΔME = - 52.5 J; **b)** ΔME = + 46.76 J ; **c)** ΔME = - 47.12 J.

3. The angle α has the value:

a) α = 23.6°; **b)** α = 21.2°; **c)** α = 22.1°.

4. The value F of the friction force is:

a) F = 21.25 N ; **b)** F = 23.56 N ; **c)** F = 25.72 N.

B- Horizontal Motion

During the motion phase between points B and C, (S) leaves B with a speed $v_B = 3.8\ m \cdot s^{-1}$ and slides until it stops at point C. Thus, (S) is subjected to a friction force $\vec{f}$ of constant magnitude f = 30 N.

1. The distance between points B and C is then:

a) $BC = \frac{v_B}{2f}$; **b)** $BC = \frac{{v_B}^2}{2f}$; **c)** $BC = \frac{m{v_B}^2}{2f}$

2. The value of the distance BC is:

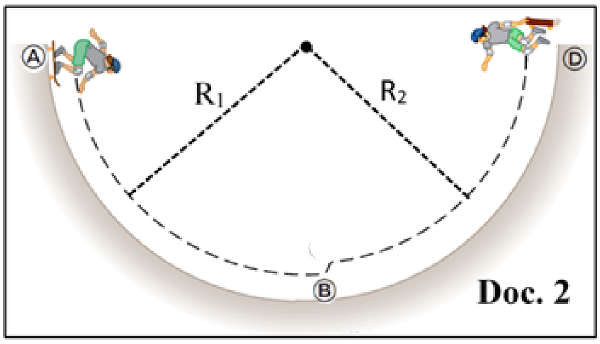
a) BC = 18.3 m ; **b)** BC = 16.5 m ; **c)** BC = 14.7 m.

C- Motion in a half-cylinder

(S) can now move in a half-cylinder with a horizontal axis of Doc. 2. (S) starts from rest at point A, at the instant $t_0 = 0$, in a crouching position at the edge of the half-cylinder. During the descent, (S) slides without friction, so that G moves along an arc of a circle of radius $R_1 = 6.30$ m. (S) passes, at the instant t_B-, at the bottom, just before point B. Take the horizontal plane passing through B as the reference level for the gravitational potential energy.

1. The speed of (G) at the bottom of the trajectory, just before point B is:

a) $V_{B^-} = 10.2$ m/s **b)** $V_{B^-} = 11.11$ m/s **c)** $V_{B^-} = 12.7$ m/s.



Doc. 2

2. Just after passing point B, at the instant t_B+, he stands up and raises his arms, thus raising his center of mass G by 0.45 m above point B and G moves along an arc of a circle of radius $R_2 = 5.85$ m.

2.1. The physical quantity, related to the motion, which remains conserved between the instants t_B- and t_B+ is:

a) The mechanical energy; **b)** The linear momentum;
c) The angular momentum.

2.2. The speed of (S) at the instant t_B+ is:

a) $V_B+ = V_B-$; **b)** $V_B+ = \frac{R_2}{R_1} V_B-$; **c)** $V_B+ = \frac{R_1}{R_2} V_B-$.

2.3. The value of the speed V_B+ is:

a) V_B+ = 11,96 m/s; **b)** V_B+ = 11,62 m/s; **c)** V_B+ = 10,97 m/s

2.4. The excess in kinetic energy is:

a) $\Delta E_C = 735$ J ; **b)** $\Delta E_C = 703$ J ; **c)** $\Delta E_C = 745$ J.

2.5. When he rises at point B, the form of energy that is converted into kinetic energy is:

a) gravitational potential energy; **b)** chemical energy of the muscles; **c)** thermal energy.

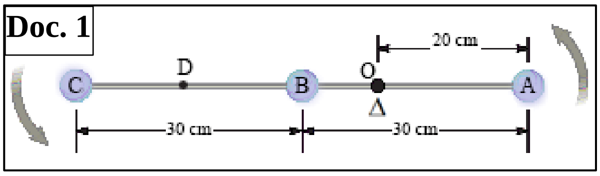
3. Then, (S) slides upward, covering the other quarter of the circle, its center of mass G moving along an arc of a circle of radius $R_2 = 5.85$ m. Passing through point D, the far edge of the half-cylinder, (S) becomes horizontal and moves at a speed of value V_D:

a) $V_D = 5.33$ m/s ; **b)** $V_D = 5.74$ m/s ; **c)** $V_D = 5.14$ m/s

4. G rises a height h above point D of value:

a) h = 1.21 m; **b)** h = 1.45 m; **c)** h = 1.67 m.

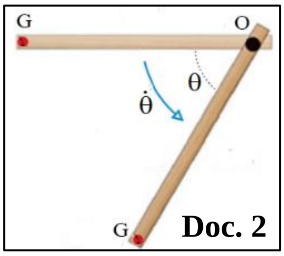
Exercise 3: Rotating Solid (7 pts)



Three particles (A), (B) and (C), each with a mass m = 200 g, are fixed to a rigid rod of negligible mass (Doc 1), thus forming a solid (S) = [rod, (A), (B), (C)]. (S) can rotate in a vertical plane around a horizontal axis (Δ), perpendicular to the rod and passing through point O. (S) starts from rest, at the instant $t_0 = 0$, in a horizontal position. During its rotation, at an instant t, (S) is subjected to a frictional force couple whose moment is constant of value $M_f = -0.10$ m·N. Take g = 10 m/s^2 and the horizontal plane passing through O as the reference level for the gravitational potential energy.

1. Let G be the center of mass of (S) and I be the moment of inertia of (S), with respect to (Δ). Then:

a) G coincides with B and I = 0.126 kg.m^2; **b)** G coincides with B and I = 0.042 kg.m^2;
c) G coincides with D and I = 0.126 kg.m^2.



2. At an instant t, the angular elongation of (S) is θ and its angular velocity is $\dot{\theta} = \frac{d\theta}{dt}$.

2.1. The expression of the mechanical energy ME of the system [(S), Earth)] at an instant t is:

a) ME = ½ I $\dot{\theta}^2$ + 3mg OG cos θ; **b)** ME = I $\dot{\theta}^2$ - 3mg OG sin θ;
c) ME = ½ I $\dot{\theta}^2$ - 3mg OG sin θ.

2.2. The variation ΔME of the mechanical energy of the system [(S), Earth)] between the instants 0 and t is, in the SI:

a) ΔME = 0.70×θ ; **b)** ΔME = - 0.60×θ ; **c)** ΔME = - 0.10×θ.

2.3. The angular velocity $\dot{\theta}$ then verifies, in the SI, the relation:

a) ½ I $\dot{\theta}^2$ +3mg OG cos θ = 0.70×θ; **b)** ½ I $\dot{\theta}^2$ - 3mg OG sin θ = - 0.10×θ ; **c)** I $\dot{\theta}^2$ - 3mg OG sin θ = - 0.60×θ

3.1. The resulting moment ΣM_Δ of the external forces applied to (S), at a date t, has the value, in the SI:

a) ΣM_Δ = 3m g OG sin θ - 0.10; **b)** ΣM_Δ = 3mg OG cos θ - 0.10 ; **c)** ΣM_Δ = 3mg OG cos θ + 0.10.

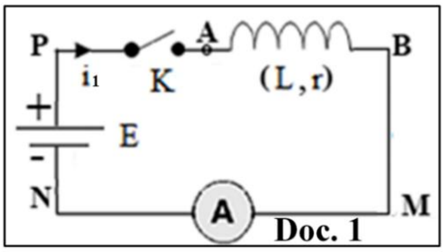
3.2. The value of θ for which $\Sigma M_\Delta = 0$ is :

a) θ = 80.4º ; **b)** θ = 85.6º ; **c)** θ = 90.0º.

3.3. The maximum value $\dot{\theta}_{max}$ 0f $\dot{\theta}$ is written as:

a) $\dot{\theta}_{max}$ = 4.59 rad/s ; **b)** $\dot{\theta}_{max}$ = 4.64 rad/s ; **c)** $\dot{\theta}_{max}$ = 5.35 rad/s.

Exercise 4: Principle of a car engine ignition (10 pts)

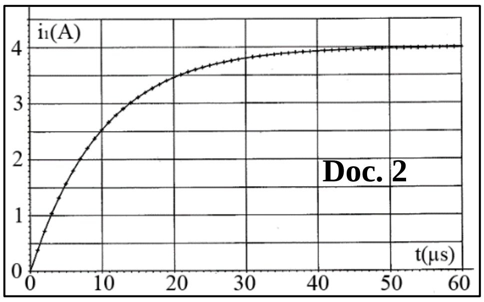


*The ignition of the air-fuel mixture in a car engine is caused by a spark that appears between the terminals of a spark plug. This spark appears when the absolute value of the voltage across the spark plug is greater than **10,000 V**.*

The formation of this spark is linked to the closing and then opening of a circuit comprising an ideal battery of e m f E = 12 V, a coil (1) of inductance L and internal resistance r and a switch K (Doc 1).

A- The switch is closed

At the instant $t_0 = 0$, we close the switch K. We record the different currents i_1 at different instants. We obtain the graph in (Doc 2).



1. The value of the internal resistance r is:

a) r = 30 Ω ; **b)** r = 3 Ω : **c)** r = 0.3 Ω.

2.1. The differential equation governing the time evolution of i_1 is:

a) $\frac{di_1}{dt} + \frac{r}{L} i_1 = E$; **b)** $\frac{di_1}{dt} + \frac{r}{L} i_1 = 0$; **c)** $\frac{di_1}{dt} + \frac{r}{L} i_1 = \frac{E}{L}$.

2.2. Let I_0 be the steady state current and $\tau = \frac{L}{r}$. The solution to the differential equation can be written as:

a) $i_1 = I_0 (1 - e^{-t/\tau})$; **b)** $i_1 = I_0 e^{-t/\tau}$; **c)** $i_{11} = I_0 (1+ e^{-t/\tau})$.

2.3. The value of the time constant τ is:

a) τ = 5 µs ; **b)** 20 µs ; **c)** 10 µs.

2.4. The value of the inductance L of the coil (1) is:

a) L = 3×10^{-5} H ; **b)** L = 3×10^{-4} H ; **c)** L = 3×10^{-6} H.

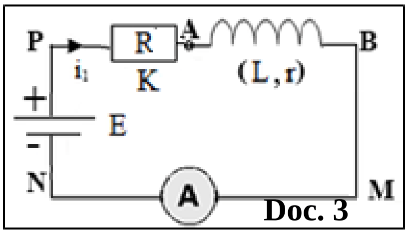
3. The maximum energy W_m stored by the coil (1) is:

a) W_m = 2.4×10^{-5} J ; **b)** W_m = 2.4×10^{-4} J ; **c)** W_m = 2.4×10^{-3} J.

B- Study of Spark Formation

This coil (1) is put near to another coil (2), placed in a separate circuit with a spark plug. According to the electromagnetic induction phenomenon, the voltage u_2 across coil (2), then across the spark plug, is related to the current i_1 carried by the coil (1), by the relationship: $u_2 = \alpha \frac{di_1}{dt}$, where α is a positive constant.

When we open the switch, at an instant chosen as the origin of time, $t_0 = 0$, a spark appears across its terminals. The air then becomes conductive and behaves like a resistor (R) of resistance R of several megohms. The circuit of coil (1) can then be modeled according to the diagram in (Doc 3).



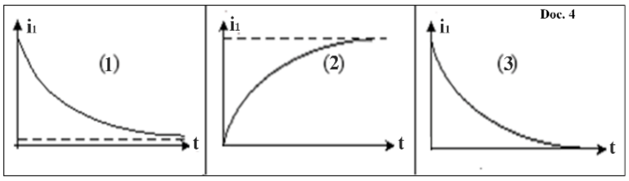
1. The differential equation in i_1, at an instant t, is:

a) $L\frac{di_1}{dt} + (r + R)\, i_1 = E$; **b)** $L\frac{di_1}{dt} + r\, i_1 = 0$; **c)** $L\frac{di_1}{dt} + R\, i_1 = E$.

2. With $\tau = \frac{L}{R+r}$, the expression, as a function of time, of the current i_1 is:

a) $i_1 = \frac{E}{R+r}(1 - e^{-\frac{t}{\tau}})$; **b)** $i_1 = \frac{E}{(r+R)} + \left(I_0 - \frac{E}{R+r}\right).e^{\frac{-t}{\tau}}$; **c)** $i_1 = I_0 - \frac{E}{R+r}(1 - e^{-\frac{t}{\tau}})$.

3. Among the three curves in (Doc 4), the curve representing the possible shape of the evolution of i_1 as a function of time is:



a) (1) ; **b)** (2) ; **c)** (3).

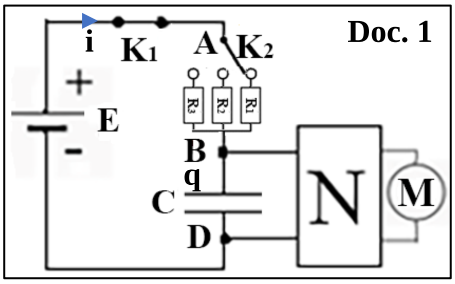
4. Experimentally, we found that the absolute value of the voltage u_2, defined above, is given by: $u_2 = 15000\, e^{-500\,t}$ (u_2 in V and t in s). The instant starting from which we can consider that there is no longer a spark across the spark plug is:

a) $t = 8.11\times10^{-4}$ s ; **b)** $t = 8.11\times10^{-5}$ s ; **c)** $t = 8.19\times10^{-4}$ s.

Exercise 5: Operating principle of intermittent car wipers (9 pts)

We study the operating principle of intermittent wipers in light rain, i.e, the automatic shutdown of the wiper motor after each wipe.

The circuit setup consists of a battery of emf E = 12 V, two switches K_1 and K_2, three resistors (R_1), (R_2), and (R_3) with respective resistances R_1, R_2, and R_3, a capacitor (C) of capacitance C = 10 mF, and an electronic sensor (N). Intermittent wipers use an adjustable resistor in an RC circuit to set the delay between two successive passes of the wipers. When we close the switch, (C), initially discharged, begins to charge, and the voltage $u_C = u_{BD}$ across its terminals increases. The sensor (N) captures this voltage u_C; and when u_C reaches the given value $U_C = 10$ V, the sensor triggers the wipers (Doc 1). Another component of the circuit, not shown, discharges (C) at this instant, allowing the cycle to start again.



At the instant $t_0 = 0$, (C) being discharged, we close switch K_1 and switch K_2 is closed on (R_1). At an instant t, the armature (B) carries a charge q, where $q = C\cdot u_C$ and the circuit carries a current i.

1. The differential equation giving the variations of u_C as a function of time is of the form:

a) $R_1 u_C + \frac{Cdu_C}{dt} = E$; **b)** $R_1 u_C + \frac{du_C}{Cdt} = E$; **c)** $u_c + R_1C\frac{du_c}{dt} = E$.

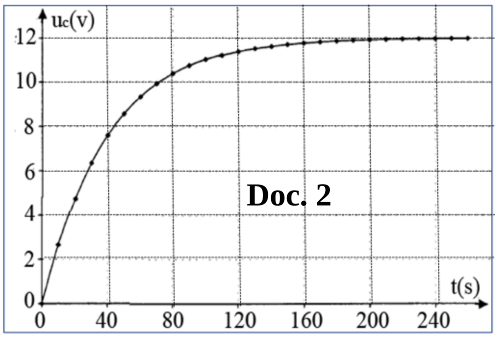
2. The solution to the previous differential equation is $u_C = A - B\cdot e^{\frac{-t}{\tau}}$ with:

a) A = B = E et $\tau = R_1C$; **b)** A = - B = E et $\tau = R_1C$; **c)** A = B = E et $\tau = \frac{1}{R_1C}$

3. The steady-state value of u_C is:

a) u_C =10 V; **b)** u_C =12 V; **c)** u_C =0 V.

4. The graphical representation of u_C is given in (Doc 2).



4.1. The windshield wipers will be activated after a time delay Δt:

a) Δt ≈ 85 s ; **b)** Δt ≈ 60 s ; **c)** Δt ≈ 70 s.

4.2. The value of τ is:

a) τ =40 s ; **b)** τ =30 s ; **c)** τ =20 s :

4.3. The value of R_1 is:

a) R_1 ≈ 3 kΩ ; **b)** R_1 ≈ 4 kΩ. **c)** R_1 ≈ 3,35 kΩ.

5. The switch (K_2) is set to (R_2); the value of R_2 for which the wipers will turn on every 30 s is:

a) R_2 ≈ 1,67 kΩ : **b)** R_2 ≈1,95 kΩ ; **c)** R_2 ≈ 2,14 kΩ.

6. The switch (K_2) is set to (R_3), of value R_3 = 560 Ω; the windshield wipers will be triggered at each delay Δt:

a) Δt ≈ 60 s ; **b)** Δt ≈ 40 s ; **c)** Δt ≈ 10 s.

Lebanese University
Faculty of Engineering
Answer sheet (A)

Entrance exam 2025-2026
July 2025
Lebanese Bac (A)

Put an X or ✓ in the corresponding box

Exercise 1 : The Age of the Earth (21 pts)

Exercise 2 : An Olympic Practice (14 pts)

Question	a)	b)	c)
A-1		X	
A-2			X
A-3	X		
A-4		X	
B-1			X
B-2	X		
C-1		X	
C-2.1.			X
C-2.2.			X
C-2.3.	X		
C-2.4.			X
C-2.5.		X	
C-3.	X		
C-4.		X	

Exercise 3 : Rotating Solid (7 pts)

Question	a)	b)	c)
1.		X	
2.1.			X
2.2.			X
2.3.		X	
3.1.		X	
3.2.	X		
3.3.		X	

Exercice 4 : Principle of Car Engine Ignition (10 pts)

Question	a)	b)	c)
A-1.		X	
A-2.1.			X
A-2.2.	X		
A-2.3.			X
A-2.4.	X		
A-3.		X	
B-1.	X		
B-2.		X	
B-3.	X		
B-4.	X		

Exercice 5 : Operating Principle of Intermittent Car Wipers (8 pts)

Question	a)	b)	c)
1.			X
2.	X		
3.		X	
4.1.			X
4.2.	X		
4.3.		X	
5.	X		
6.			X

Université Libanaise **Corrigé concours d'entrée 2025-2026** **Juillet 2025**
Faculté d'ingénierie **Physique (A)** **Durée 120 min**

Afin d'éliminer les stratégies de réponses au hasard, **chaque bonne réponse est gratifiée de 1,25 point**, tandis que **la mauvaise réponse est pénalisée** par le retrait **de 0,5 point**.

Prendre, au besoin, $g = 10\ m/s^2$.

Exercise 1: The Age of the Earth (21 pts)

A- Decay of the uranium 238 nucleus (6 ½)

1. The gamma emission is not taken into account.

1.1the reaction of this decay.

${}^{238}_{92}U \rightarrow {}^{A}_{Z}Th + \alpha$; avec $\alpha = {}^{4}_{2}He$

Conservation of mass number: $238 = A + 4 \Rightarrow A = 234$.

Conservation charge number: $92 = Z + 2 \Rightarrow Z = 90$.

${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$. (1½)

1.2. The mass loss Δm is given by: $\Delta m = m({}^{238}_{92}U) - (m({}^{234}_{90}Th) + m({}^{4}_{2}He))$

$\Delta m = 238.0003 - (233.9942+4.0015) \Rightarrow \Delta m = 4.6\times10^{-3}$ u. (1)

1.3. The energy released during this decay is:

$E = \Delta m\times c^2 = 4.6\times10^{-3}\times 931.5 = 4.2849$ MeV. (1)

1.4.1. Conservation of the total energy:

$m(U)\times c^2 = m(Th)\times c^2 + KE(Th) + m(\alpha)\times c^2 + KE(\alpha)$

$\Leftrightarrow \Delta m\times c^2 = KE(Th) + KE(\alpha) = 4.2849$ MeV. (1)

1.4.2. We have $KE(Th) + KE(\alpha) = 4.2849$ MeV $\Leftrightarrow (0.0171+1)\ KE(\alpha) = 4.2849$

$\Rightarrow KE(\alpha) = \frac{4.2849}{1.0171} = 4.2129$ MeV.

The kinetic energy of the emitted particle : $KE(\alpha) = 4.2129$ MeV.

The kinetic energy of the daughter nucleus is: $KE(Th) = 4.2849 - 4.2129 = 0.0720$ MeV. (2)

2. The gamma emission is taken into account (4 ½)

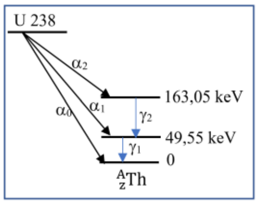
2.1. he de-excitation reaction of the excited thorium nucleus:

${}^{238}_{92}U \rightarrow {}^{234}_{90}Th^* + {}^{4}_{2}He$; With: ${}^{234}_{90}Th^* \rightarrow {}^{234}_{90}Th + \gamma$;

So: ${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He + \gamma$

$E = \Delta m\times c^2 = KE(Th) + KE(\alpha) + E_{exc} \Rightarrow$

$KE(Th) + KE(\alpha) + E_{exc} = 4.2849$ MeV.



The kinetic energy of the α_0 particle is when the daughter nucleus is in the ground state E_0. Thus: $KE(Th) + KE(\alpha_0) + E_0 = 4.2849$ MeV ; $KE(\alpha_0) = 4.2129$ MeV.

The kinetic energy of the α_1 particle is when the daughter nucleus is in the excited state E_1: $KE(\alpha_1) + KE(Th) + E_1 = 4.2849$ MeV

So : $KE(\alpha_1) = 4.2849 - 0.04955 - 0.0720 = 4.16335$MeV.

The kinetic energy of the α_2 particle is when the daughter nucleus is in the excited state E_2:

$KE(\alpha_2) + KE(Th) + E_2 = 4.2849$ MeV

$KE(\alpha_2) = 4.2849 - 0.16305 - 0.0720 = 4.04985$ MeV. (2½)

2.2. $E_1(\gamma_1) = 49.55 - 0 = 49.55$ keV and $E_1 = \frac{h\,c}{\lambda_1} \Rightarrow \lambda_1 = \frac{h\,c}{E_1} = \frac{6.63\times10^{-34}\times3\times10^{8}}{49.55\times10^{3}\times1.6\times10^{-19}} = 2.51\times10^{-11}$ m.

$E_2(\gamma_2) = 163.05 - 49.55 = 113.5$ keV and $E_2 = \frac{h\,c}{\lambda_2} \Rightarrow \lambda_2 = \frac{h\,c}{E_2} = \frac{6.63\times10^{-34}\times3\times10^{8}}{113.5\times10^{3}\times1.6\times10^{-19}} = 1.10\times10^{-11}$ m. (2)

3. Nucleus Stability (4)

3.1. The mass defect of the uranium 238 nucleus is:

$\Delta m = Z\times m_P - (A-Z)\times m_n - m({}^{238}_{92}U) = 92\times1.0073 + 146\times1.0087 - 238.0003 = 1.9415$ u

The binding energy of the uranium nucleus is: $E_b = \Delta m\times c^2 = 1.9415\times931.5 = 1808.50725$ MeV. (2)

3.2. The binding energy per nucleon of the uranium 238 nucleus is: $E_b/A = \frac{1808.50725}{238} \approx 7.60$ MeV/nucleon.

The binding energy per nucleon of the thorium 234 nucleus is: $E_b/A = \frac{1784.38}{234} \approx 7.63$ MeV/nucleon.

So the thorium nucleus is more stable than the uranium nucleus because:

$E_b/A(Th) = 7.63$ MeV/nucleon > $E_b/A(U) = 7.60$ Me V/nucleon. (2)

II. Geochronology

1. ${}^{238}_{92}U \rightarrow {}^{206}_{82}Pb + x({}^{0}_{-1}e) + y\ ({}^{4}_{2}He)$.

Conservation of mass number: 238 = 206 + 4 y $\Rightarrow$ y = 8 (α decay).

Conservation of charge number: 92 = 82 -x + 2y $\Rightarrow$ x = 6 (β^- decay). (1)

2.1. We have : $N_U(t)=B\cdot e^{\frac{-t}{\tau}}$. At the instant $t_0 = 0$, $N_U(0) = B\cdot e^0 \Rightarrow B = N_U(0)$.

The activity of the specimen is written as: $A = -\frac{dN_U}{dt} = \frac{B}{\tau}\times e^{\frac{-t}{\tau}} = \lambda.N_U$

By replacing, we obtain:

$-\frac{B}{\tau}\times e^{\frac{-t}{\tau}} + \lambda\, B\cdot e^{\frac{-t}{\tau}} = 0 \Rightarrow \forall t,\ \tau = \frac{1}{\lambda}$. (1½)

2.2. For t = τ, $N_U = 0.37\, N_U(0) = 0.37\times5\times10^{12} = 1.85\times10^{12}$ nuclei $\Rightarrow \tau = 6.4\times10^9$ years. $\lambda = \frac{1}{\tau} = \frac{1}{6{,}4\times10^9}$

$\lambda = 1.56\times10^{-10}$ year^{-1}. (1½)

2.3. To each disintegration of a U 238 nucleus corresponds to the production of a lead nucleus Pb 206 $\Rightarrow$ $N_U(t_{Earthe}) = N_U(0) – N_{Pb}(t_{Earth}) = 5\times10^{12} – 2.5\times10^{12} = 2.5\times10^{12}$ nuclei. (1)

2.4. Determining the age t_{Earth} of the Earth:

We have: $N_U(t) = N_U(0)\cdot e^{\frac{-t}{\tau}} \Leftrightarrow 2.5\times10^{12} = 5\times10^{12}\cdot e^{-\lambda t} \Rightarrow 0.5 = e^{-\lambda t}$

$\Rightarrow \ln(0.5) = -\lambda t_{Earth} \Leftrightarrow -0.693 = -\lambda\, t_{Earthe}$.

The age t_{Earth} of the Earth: $t_{Earth} = \frac{0.693}{1.56\times10^{-10}} = \ = 4.44\times10^9$ years. (1)