LEBANESE UNIVERSITY
FACULTY OF ENGINEERING

الجامعة اللبنانية
كلية الهندسة

Entrance Exam 2004-2005 **Duration: 2 hours**

Physics

I- [6 pts] Charging and discharging of a capacitor

In order to study the charging and the discharging of a capacitor, we consider the circuit on the adjacent figure, where $u_{PN} = E$ = constant and $r = 1\ k\Omega$.

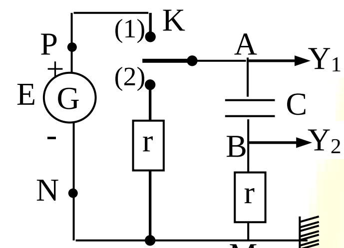


Fig. 1

A- Charging of the capacitor

At the instant $t_0 = 0$, we put the switch (K) in position (1).

1. Establish the differential equation that is verified by the voltage $u_C = u_{AB}$.

2. The solution of this equation is of the form: $u_C = D\,(1 - e^{-\frac{t}{\tau}})$. Deduce the expressions of D and τ in terms of r, C and E.

3. a. The waveform of figure 2, giving the variations of u_C in terms of time, is obtained by pushing the button « INV » of channel Y_2 and the button « ADD ». Justify.

b. Using this waveform, determine E and C.

4. Determine the instantaneous expression of the current i. Draw then the shape of the voltage u_{BM}.

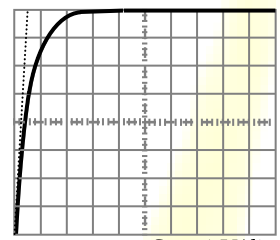


Fig. 2 $S_V = 1$ V/div
$S_h = 2$ ms/div

B- Discharging of the capacitor

The capacitor is completely charged and the buttons « INV » of channel Y_2 and « ADD » are always pushed. At the instant $t_0 = 0$, we put the switch into position (2). We obtain the waveform of figure 3 which represents the variations of $u_C = u_{AB}$ as a function of time.

1. The variations of u_C is given by: $u_C = E\,e^{-\frac{t}{\tau'}}$. Determine the expression of τ'. Verify the answer using the waveform of figure 3.

2. Draw the shape of the voltage u_{BM} and specify the used scale.

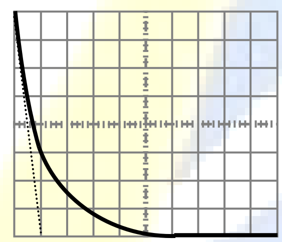


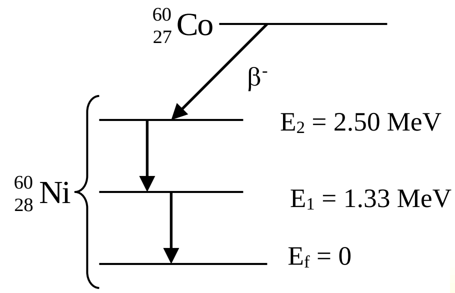
Fig. 3 $S_V = 1$ V/div
$S_h = 2$ ms/div

II- [6 pts] the cobalt 60

The cobalt 60 « $^{60}_{27}Co$ », a radioelement that is used in different applications (cobalt « bomb »), is produced when the natural cobalt $^{59}_{27}Co$ (stable isotope), the nickel $^{60}_{28}Ni$ or the copper $^{63}_{29}Cu$ are bombarded with neutrons.

A- Production and disintegration of cobalt 60

1. Write the three equations of the reactions that produce the cobalt 60.
2. The nucleus of the cobalt 60 is transformed, by β^- emission, into a daughter nucleus ${}^{60}_{28}Ni$ in an excited state of energy E_2 = 2.50 MeV. ${}^{60}_{28}Ni$ returns to the fundamental state, of energy E_f = 0, into 2 steps, which correspond to the emission of 2 photons (see figure).



a. Calculate the energy that is liberated by this disintegration.
b. The kinetic energy E_C of the β^- particle is not quantized. Why?
c. Calculate the wavelengths of the radiations associated with the two photons.

B- Radioactive decay of cobalt 60

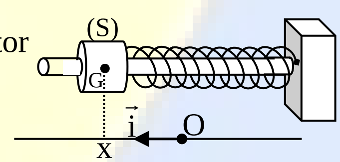
We study a sample containing only, at the instant $t_0 = 0$, cobalt 60 of mass m_0 = 1 mg. Each year, we measure the activity A of this sample. We notice that the ratio $\frac{A(t)}{A(t+1)}$ has an average value of 1.14, where A(t) and A(t+1) are respectively the activity of the sample at a given instant t and at the instant (t+1), that is one year later, t being expressed in years. Let A_0 be the activity of the sample at the instant $t_0 = 0$.

1. Give the definition of the activity and write the expression of the activity A(t).
2. Determine the radioactive constant λ.
3. Calculate the time after which the activity becomes $\frac{A_0}{2}$. What does this time represent?
4. Calculate the mass of ${}^{60}_{27}Co$ that is disintegrated after one year.

Given: $h = 6.63\times10^{-34}$ J.s ; $c = 3\times10^{8}$ m/s ; 1 eV = 1.6×10^{-19} J; m(${}^{60}_{27}Co$) = 59.9190 u ; m(${}^{60}_{28}Ni$) = 59.9154 u; m(${}^{0}_{-1}e$) = 5.5×10^{-4} u; 1 u = 1.66×10^{-27} kg = 931.5 MeV/c^2.

III- [8 pts] Mechanical oscillator

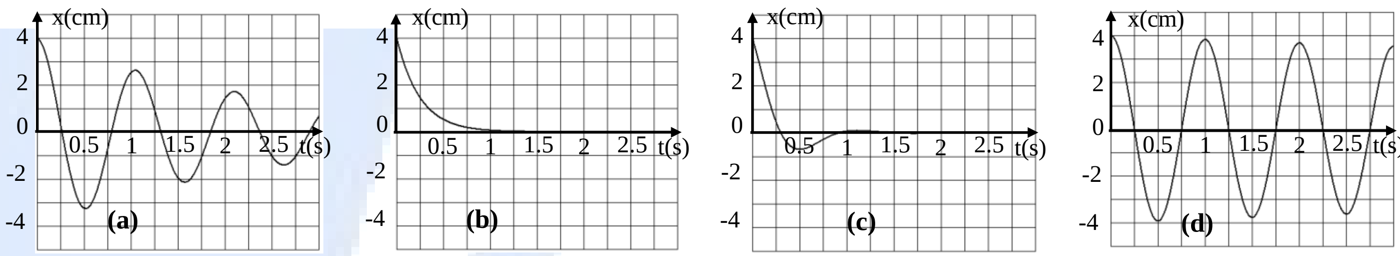
The purpose of this exercise is to study the behavior of a horizontal mechanical oscillator as a function of the magnitude F of the force of friction. A horizontal mechanical oscillator is made of a solid (S), of mass m = 0.635 kg, that is fixed at the free end of a spring (R) of negligible mass and of spring constant (stiffness) k = 25.0 N/m.



At any instant t, the abscissa x of the center of inertia G of (S) is located with respect to a horizontal axis (O, $\vec{i}$), where O is the abscissa of G at equilibrium and $\dot{x}$ = V, the algebraic value of the velocity of (S). The horizontal plane containing G is taken as the reference level of the gravitational potential energy.

A- Experimental study

We displace (S), from its equilibrium position, 4,00 cm to the left and we release it without initial velocity at the instant $t_0 = 0$. An appropriate device allows us to visualize the motion of (S) for different values of F of the force of friction exerted on (S). (See figures (a), (b), (c), and (d)).



1. Let us consider the case of figure (a). Calculate the loss of energy that the system [(S), spring] undergoes after the first oscillation. Deduce the average value of the force of friction that is supposed constant during this first oscillation.
2. a. Arrange, with justification, the sketches with respect to the increasing values of F.
   b. What do we obtain if we eliminate the force of friction?
   c. Which type of motion does (S) perform when it does not oscillate?
3. How does the duration of an oscillation vary when F increases? How do we call the duration of such oscillation?
4. In which case can we consider that the duration of an oscillation is almost equal to the natural (proper) period T_0 of the oscillator? Why?

B- Theoretical study

(S), that is displaced 4,00 cm to the right from its equilibrium position, is launched, at the instant $t_0 = 0$, with an initial velocity $\vec{V}_0 = V_0 \vec{i}$ where $V_0 = 0.281$ m/s. (S) begins then to oscillate without friction around its equilibrium position.

1. By applying the law of conservation of the mechanical energy of the system [(S), spring]:
a. find the value x_m of the amplitude of oscillations of (S).
b. determine the second order differential equation that governs the motion of (S) and calculate T_0. The obtained value is in agreement with the experiment. Justify.
2. The solution of this equation is of the form: $x = x_m \cos(\frac{2\pi}{T_0} t + \phi)$. Show that ϕ can have the value -2,30 rd.
3. Calculate the time t_1 after which (S) passes by O for the first time. Draw the shape of x in terms of t.

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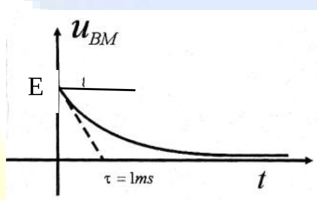
<u>Solution of Physics</u>

Expected answers	Scale	Comments
<u>Exercise 1- Charging and discharging [6 pts]</u>		
A- 1- Knowing that $i = \frac{dq}{dt}$ and $q = C.u_C$; and $E = u_C + ri$ by substitution We get $E = u_C + rC\frac{du_C}{dt} \Leftrightarrow \frac{du_C}{dt} + \frac{1}{rC}u_C = \frac{E}{rC}$ differential equation of second order without second member.		
2- the solution of the previous differential equation is $u_C = D(1 - e^{-\frac{t}{\tau}})$. By deriving this equation with respect to time we get: $\frac{du_C}{dt} = \frac{D}{\tau}e^{-\frac{t}{\tau}}$; by substitution in the differential equation we get $\frac{D}{\tau}e^{-\frac{t}{\tau}} + \frac{D}{rC} - \frac{D}{rC}e^{-\frac{t}{\tau}} = \frac{E}{rC}$ we can prove that $\tau = rC$ and $E = D$		
3- a- $u_{AM} = u_{AB} + u_{BM} \Leftrightarrow u_{AB} = u_{AM} - u_{BM}$ (INV for u_{BM} and to get u_{AB}) E b- From the figure when $t \to \infty$; $E = (1\text{ V/div} \times 8\text{ div}) = 8$ V $\tau = rC = (½\text{ div} \times 2\text{ms/div}) = 1$ ms $\Leftrightarrow C = 10^{-6}$ F		
4- $i = C\frac{du_C}{dt} = C\frac{D}{\tau}e^{-\frac{t}{\tau}} = C\frac{E}{rC}e^{-\frac{t}{\tau}} = \frac{E}{r}e^{-\frac{t}{\tau}}$. then $u_{BM} = ri = E\,e^{-\frac{t}{\tau}} = 8\times e^{-1000t}$  		
B- **Discharging of the capacitor** 1- $u_C = E\,e^{-\frac{t}{\tau'}}$; $u_{AM} = ri = u_C + (-ri) \Rightarrow$ $-2ri + u_C = 0$; and $i = -\frac{dq}{dt}$ (discharging) $= -C\frac{du_C}{dt}$ $\Rightarrow 2rC\frac{du_C}{dt} + u_C = 0 \Rightarrow \tau'\frac{du_C}{dt} + u_C = 0 \Rightarrow \tau' = 2rC$. From the figure 3. The tangent to the curve at the origin cuts the the time axis at t = 1div $\Rightarrow \tau' = 1\text{div} \times 2\text{ms/div} = 2\text{ ms} = 2\tau$.		

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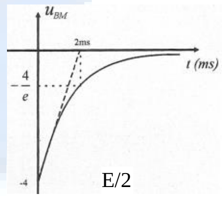
2- u_{BM} = - ri and i = $-C\,\frac{du_C}{dt}$  $= -C\left(-\frac{E}{\tau'}e^{-\frac{t}{\tau'}}\right) = +\frac{C}{\tau'}u_C; \Rightarrow$ $u_{BM} = -\frac{rC}{\tau'}u_C = -\frac{\tau}{\tau'}u_C = \frac{1}{2}u_C$		

Expected answers	Scale	Comments
<u>Second exercise: The cobalt 60 [6 pts]</u> **A-1-** The production of cobalt 60 is obtained: a- ${}^{1}_{0}n + {}^{59}_{27}Co \rightarrow {}^{60}_{27}Co + \gamma$; b- ${}^{1}_{0}n + {}^{60}_{28}Ni \rightarrow {}^{60}_{27}Co + {}^{1}_{1}H$ c- ${}^{1}_{0}n + {}^{63}_{29}Cu \rightarrow {}^{60}_{27}Co + {}^{4}_{2}He$ 2- ${}^{60}_{28}Co \rightarrow {}^{60}_{28}Ni + {}^{0}_{-1}e(\beta^{-}) + {}^{0}_{0}\nu$. a- The mass defect: $\Delta m = [m(Co) - m(Ni) - m(\beta^{-}) = 0.00305\ u]$. The liberated energy for this reaction is given by : $E = \Delta m\ c^2$ $= 0.00305 \times 931.5 = 2.84$ MeV b- The β^{-} decay is accompanied with the emission of an antineutrino which can take any value. Since the kinetic energy of the particle β^{-} is not quantized. c- The two emitted photons have the respective energy: 1st photon is $E_1 = 1.33$ MeV; but $E_1 = h\nu_1 = \frac{hc}{\lambda_1} \Rightarrow \lambda_1 = \frac{hc}{E_1}$ $= 9.35\times10^{-13}$ m. The second Photon $E' = E_2 - E_1 = 2.50 - 1.33 = 1.17$MeV $\Rightarrow$ $E' = h\nu_2 = \frac{hc}{\lambda_2} \Rightarrow \lambda_2 = \frac{hc}{E'} = 1.06\times10^{-12}$m. B-1- The activity is the number of disintegrations per unit of time $A(t) = A_0 e^{-\lambda t}$. 2- $A(t) = A_0 e^{-\lambda t}$ and $A(t+1) = A_0\ e^{-\lambda(t+1)} \Rightarrow \frac{A(t+1)}{A(t)} = \frac{1}{e^{-\lambda}} = e^{\lambda} = 1.14$ $\Rightarrow \lambda = Ln(1.14) = 0.131$ year^{-1}. 3- $A(t_1) = \frac{A_0}{2} = A_0\ e^{-\lambda t} \Rightarrow e^{-\lambda t} = 2 \Rightarrow t_1 = \frac{Ln2}{\lambda} \Rightarrow t_1 = 5.2$ years. t_1 represents the radioactive period or half-life. 4- the disintegrated mass in one year : $m_d = m_0 - m(1year) = m_0\ (1 - e^{-\lambda\ 1})$ $= 1(1 - 0.877) = 0.123$ mg		

Expected answers	Scale	Comments
<u>Third exercise: Mechanical oscillator [8 pts]</u> **A- Experimental study** 1- At the time $t_0 = 0$: the amplitude $X_0 = 4$ cm; the mechanical energy = the potential energy = $\frac{1}{2}\,K\,X_1^2$. The energy lost = $\frac{1}{2}k\,X_1^2 - \frac{1}{2}\,k\,X_0^2 =$ $\frac{1}{2}[25(7.0225 - 16) = -1.12\times10^{-2}$ J < 0 energy lost. The variation of mechanical energy: $\Delta ME = W(\vec{F}) = -F_{ave}\,.\,\ell$ $= -1.12\times10^{-2}$. With $\ell = 4 + 3.2\times2 + 2.65 = 13.05 = 0.1305$ m $\Rightarrow F_{av} = 8.58 \times 10^{-2}$N. 2- a- d; a; c: b. + justification d- Undammed oscillations. e- Non periodic motion f- 3- When F increases, the time of one oscillation increases. ($T_a > T_d$). in this case the period is called pseudo-period. 4- in the case (d) where the damping is small. We can consider that the period is approximately equal to the proper period. **B- Theoretical study** 1-a- ME = $\frac{1}{2}kx^2 + \frac{1}{2}mV^2 = \frac{1}{2}kx_0^2 + \frac{1}{2}mV_0^2 = \frac{1}{2}\,k\,x_m^2 \Rightarrow$ $x_m^2 = x_0^2 + \frac{m}{k}V_0^2 \Rightarrow x_m = 6$ cm b- $\frac{dME}{dt} = 0 \Rightarrow m\dot{x}\ddot{x} + kx\dot{x} = 0$. divide by $\dot{x} \Rightarrow \ddot{x} + \frac{k}{m}x = 0$ $\Rightarrow \ddot{x} + \omega_0^2\,x = 0$; differential equ………. $\Rightarrow \omega_0 = \sqrt{\frac{k}{m}} = \frac{2\pi}{T_0}$ $\Rightarrow T_0 = 2\pi\sqrt{\frac{m}{k}} = 1$ s. this value is verified by the figure (d) where T = 1s 3- $x = x_m\cos(\frac{2\pi}{T_0}\,t + \varphi)$; $V = \dot{x} = -x_m\frac{2\pi}{T_0}\sin(\frac{2\pi}{T_0}\,t + \varphi)$. at t =0		

$x_0 = x_m\cos\varphi$ and $V_0 = \dot{x} = -\, x_m \frac{2\pi}{T_0}\, \sin\varphi$ divide the two expression , we get $\tan\varphi = 1.12 \Rightarrow \varphi = -\,2.3$ rd . 4- at the time t_1; $x_1 = 0 \Rightarrow \cos(2\pi t_1 + \varphi) = 0$ and $\dot{x}(t_1) = -2\pi x_m \sin(2\pi t1 + \varphi) > 0$ $\Rightarrow 2\pi t_1 + \varphi = -\frac{\pi}{2} \Rightarrow t_1 = 0.116$ s.