LEBANESE UNIVERSITY
FACULTY OF ENGINEERING

الجامعة اللبنانية
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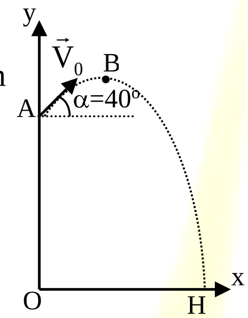
Entrance exam 2003-2004 Physics Duration: 2 hours



First exercise: Duration of fall of a diver [6 points]

A diver, considered as a particle of mass m = 80.0 kg, jumps into the water of a pool from a springboard placed at A, 6.00 m above the surface of the water. He leaves the springboard with a velocity $\vec{V}_0$, of magnitude V_0 = 5,00 m/s, making an angle α = 40,0° with respect to the horizontal. Neglect air resistance. Take g = 9.80 m/s^2.

1. Find the horizontal and vertical components P_{0x} and P_{0y} respectively of the initial linear momentum $\vec{P}_0$ of the diver.
2. By applying Newton's second law, show that at any instant t:
   a. the horizontal component P_x of the linear momentum $\vec{P}$ remains constant and equal to P_{0x},
   b. the vertical component P_y of $\vec{P}$ is of the form: $P_y = a\cdot t + b$. Determine a and b.
3. Determine the components of $\vec{P}$, at B, the top of the trajectory. Deduce the time elapsed to reach B.
4. The diver reaches the point H at the water surface considered as the reference level of the gravitational potential energy. Determine the magnitude V_H of the velocity $\vec{V}_H$ at point H, the magnitude V_{Hy} of the vertical component of $\vec{V}_H$ and the time of motion elapsed for the diver to pass from A to H.

Second exercise: Speed of a β^- particle [5 points]

The ${}^{99}_{43}Tc$ isotope, which is the subject of this exercise, is actually used in medical imaging.

It is obtained, with a molybdenum/technetium generator, from the molybdenum isotope ${}^{99}_{42}Mo$. This isotope is β^- radioactive of half-life 2.8 days.

1. Write the disintegration equation of ${}^{99}_{42}Mo$.
2. The molybdenum nucleus being initially at rest, calculate, in joule, the energy liberated by this disintegration.
3. During the disintegration of molybdenum nuclei, we find that the kinetic energy of the β^- particles is not quantized.

a. Recall the definition of « quantized energy ».
b. Tell why the β^- kinetic energy is not quantized.
c. Determine, in joule, the maximum kinetic energy of an emitted β^- particle. Using a convenient formula of the classical mechanics, find the magnitude V of the velocity of the β^- particle. What do you conclude? Give the statement of the corresponding Einstein postulate.

d. Knowing that the kinetic energy of a relativistic particle is given by:

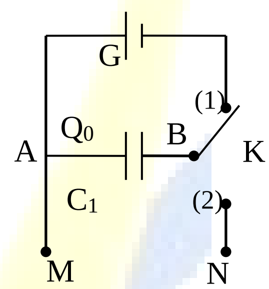
$$\text{KE (relativistic)} = m \cdot c^2(\gamma - 1) \text{ with } \frac{1}{\gamma} = \sqrt{1 - \frac{V^2}{c^2}}$$

where V is the speed of the β^-particle, m its mass and c the speed of light in vacuum, calculate V with respect to the laboratory frame of reference.

Given : mass of ($^{99}_{42}Mo$) nucleus = 98.88437 u ; mass of ($^{99}_{43}Tc$) nucleus = 98.88235 u ;
mass of (β^-) particle= 5.5×10^{-4} u = 9.11×10^{-31} kg ; 1 u = 931.5 MeV/c^2 = 1.66×10^{-27}kg ;
speed of light in vacuum c = 3×10^8 m/s ; 1 MeV = 1.60×10^{-13} J.

Third exercise: Study of some modes of discharging of a capacitor [9 points]

In order to study different modes of discharging of a capacitor, we have a generator (G) whose voltage across its terminal is constant of value U = 4.6 V, a resistor (R) of resistance R = 1 kΩ, two capacitors (C_1) and (C_2) of respective capacitances C_1 = 2.2 μF and C_2 = 4.7 μF, a coil (B) of inductance L = 75.4 mH and of negligible internal resistance, a switch (K) and connecting wires.



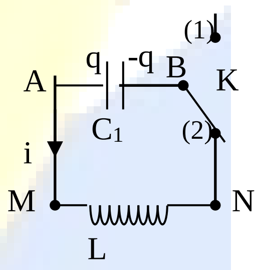
A. Charging of the capacitor

We set up the adjacent circuit where (K) is in position (1). Calculate the charge Q_0 and the electric energy stored in the capacitor (C_1).

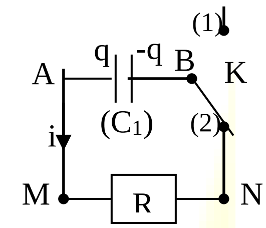
B. Three modes of discharging

I- Discharging through the coil

We connect the coil (B) between the points M and N of the previous circuit. Then we place, at the instant t = 0, the switch in the position (2).



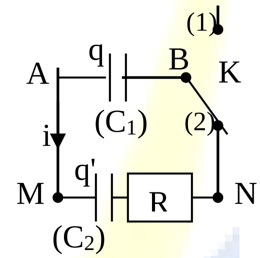
1. What is the value, at the instant t = 0, of the magnetic energy stored in the coil. Deduce the current i at t = 0.
2. Give, at any instant t, the relation between the current i and the charge q of the capacitor. Justify the answer.
3. Give, in terms of L and q, the voltage $u_{MN} = V_M - V_N$ and derive the differential equation that describes the variation of the charge q of (C_1) with respect to time.
4. The solution of this differential equation is of the form: $q = a_1\cos\omega_0 t + b_1\sin\omega_0 t$. Considering the above initial conditions, determine ω_0, a_1 and b_1.
5. Give the shape of the curve representing the variation of q in terms of time specifying two characteristic points of the graph.



II. Discharging through a resistor

We replace the coil by the resistor (R). The switch (K) is back into position (1) in order to charge again the capacitor (C_1), and then we place the switch into position (2) at t = 0.

1. Give, at any instant t, the expression of the voltage u_{MN} in terms of q and R.
2. Deduce the differential equation describing the variation of the charge q of (C_1) as a function of time t.
3. The solution of this differential equation is of the form: $q = a_2 + b_2 \cdot e^{\alpha \cdot t}$. Determine a_2, b_2 and α.
4. What do $(-1/\alpha)$ represent for the circuit?
5. Give the shape of the curve representing the variation of q as a function of time specifying two characteristic points of the graph.



III. Discharging through a capacitor in series with a resistor

We place the second capacitor (C_2) in series with (R) between M and N. (C_1) having initially the charged Q_0, (K) is placed, at t = 0, into the position (2).

At the instant t, the circuit carries a current i and (C_1) and (C_2) have respectively the charges q and q'.

1. Express the current i in terms of q'. Deduce that the charge q' is related to the charge q by : $q' = Q_0 - q$.
2. Express the voltage u_{MN} in terms of Q_0, C_1, C_2, q and R.
3. Show that the differential equation describing the variation of the charge q of the capacitor in terms of time is given by: $R\frac{dq}{dt} + q\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = \frac{Q_0}{C_2}$.
4. The solution of this differential equation is of the form: $q = a_3 + b_3 \cdot e^{\beta \cdot t}$. Determine a_3, b_3 and β.
5. Give, justifying it, the shape of the curve representing the variation of q in terms of time t specifying two characteristic points.

Entrance exam 2003-2004 **Solution of Physics** **Duration: 2 hours**

1. The vector quantity of the initial linear momentum : $\overline{P}_0 = m\vec{V}_0$

$P_{0x} = mV_{0x} = mV_0\cos40^o = 80\times5\times\cos40^o =$ **306.42 kgm/s**

$P_{0y} = mV_{0y} = mV_0\sin40^o = 80\times5\times\sin40^o =$ **257.12 kgm/s**

2. The 2nd Law of Newton: $\sum\vec{F} = \frac{d\vec{P}}{dt} = m\vec{g}$ = constant

$\Rightarrow \vec{P} = m\vec{g}\cdot t + \vec{P}_0 \Rightarrow P_x = P_{0x}$ and $P_y = -mgt + P_{0y}$.

$\Rightarrow$ **P_x = 306.42 kgm/s** and **P_y = -784 t + 257.12 kgm/s**.

$\Rightarrow$ **a = -784 kgm/s^2** and **b = 257.12 kgm/s.**

3. At point B , the value ofe P_x remains the same, but P_y becomes zero $\Rightarrow P_y = 0 \Rightarrow 0 = -784\,t + 257.12 = 0 \Rightarrow$ **t = 0.328 sec.**

4. **a**) Apply the conservation of mechanical energy :

$M.E(A) = M.E(H) \Rightarrow K.E(A) + mgh_A = K.E(H) + mgh_H$

$\Rightarrow \frac{1}{2} m V_H^2 = \frac{1}{2} m V_A^2 + mgh_A \Rightarrow 40\,V_H^2 = 1000 + 4704$

$\Rightarrow$ **V_H = 11.94 m/sec**

b) $V_{Hy} = \sqrt{V_H^2 - V_{Hx}^2} = \sqrt{(11.94)^2 - (3.83)^2} \Rightarrow$ **V_H = 11.3 m/sec**,

With $V_{Hx} = V_{0x}$.

c) $P_{Hy} = mV_{Hy} \Rightarrow -784\,t + 257.12 = 80\times11.3 \Rightarrow$ **t = 1.48 sec**.

II-

1. $^{99}_{42}\text{Mo} \longrightarrow {}^{99}_{43}\text{Tc} + {}^{0}_{-1}\text{e} + {}^{0}_{0}\bar{\nu}$;

2. $\Delta m = 98.88437-[98.88235+5.5\times10^{-4}] \Rightarrow$ **$\Delta m = 0.00147$ u $= 2.4402\times10^{-30}$ kg**.

The Liberated energy : $E = \Delta m\, c^2 = 2.4402\times10^{-30}\times9\times10^{16} \Rightarrow$ **$E = 2.19618\times10^{-13}$ J**

3. a. The energy is quantized because the values are discontinuous (discrete)

b. Because of the presence of the antineutrino

c. The maximum K.E of the emitted electron = **$E_{liberated} = 2.19618\times10^{-13}$ J**.

According to the classical mechanics : $K.E = \frac{1}{2} mV^2$

$$\Rightarrow V^2 = \frac{2K.E}{m} = \frac{2\times2.19618\times10^{-13}}{9.1\times10^{-31}} = 4.8109\times10^{17} \Rightarrow \mathbf{V = 6.936\times10^{8}\ m/sec}.$$

I conclude that V>c, Which is a contradiction of the 2nd postulate of Einstein : In Gallelian , The velocity of light in vacuum is greater than the speed of the body .

d. The relation of the K.E : $\sqrt{1-\frac{V^2}{c^2}} = \frac{1}{\frac{K.E}{mc^2}+1} = \frac{1}{\frac{2.19618\times10^{-13}}{9.1\times10^{-31}(3\times10^{8})^2}+1}$

$$\sqrt{1-\frac{V^2}{c^2}} = \frac{1}{3.68154} = 0.27165 \Rightarrow 1-\frac{V^2}{c^2} = 0.07378 \Rightarrow \frac{V^2}{c^2} = 0.92622$$

$\Rightarrow$ **$V = 0.9624$ c**.

A

1. $Q_0 = C_1 U = 2.2\times10^{-6} \times 4.6 =$ **1.012×10^{-5} C** ; $E = \frac{1}{2}CU^2 =$ **2.33×10^{-5} J**

B.

I - 1. $E_{mag.} = \frac{1}{2}CU^2 = \frac{1}{2}Li^2 = 0 \Rightarrow i = 0$

2. $i = -\frac{dq}{dt}$, Because i enters the armature of – q

3. $u_{MN} = L\frac{di}{dt} + ri$, But $r = 0 \Rightarrow u_{MN} = L\frac{di}{dt}$, $\Rightarrow u_{MN} = -L\frac{d^2q}{dt^2}$

But $u_{MN} = -L\frac{d^2q}{dt^2} = \frac{q}{C} \Rightarrow \frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$ or $\ddot{q} + \frac{1}{LC}q = 0$

4. $\frac{dq}{dt} = -\omega_0 a_1 \sin\omega_0 t + \omega_0 b_1 \cos\omega_0 t$; $\frac{d^2q}{dt^2} = -\omega_0^2 a_1 \cos\omega_0 t - \omega_0^2 b_1 \sin\omega_0 t$

$\Rightarrow \omega_0^2 = \frac{1}{LC} \Rightarrow$ **$\omega_0 = 2455$ rd/s.**

At $t = 0$, $q = Q_0 \Rightarrow Q_0 = a_1$. At $t = 0$, $i = 0 \Rightarrow b_1 = 0$.

5. Sinusoidal ; Period or Amplitude

II - 1. $u_{MN} = Ri = -R\frac{dq}{dt}$

2. $u_{MN} = \frac{q}{C} = -R\frac{dq}{dt} \Rightarrow \frac{dq}{dt} + \frac{1}{RC}q = 0.$

3. $\frac{dq}{dt} = \alpha b_2 e^{\alpha t} \Rightarrow \alpha b_2 e^{\alpha t} + \frac{1}{RC}(a_2 + b_2 e^{\alpha t}) = 0 \Rightarrow a_2 = 0$ and $\alpha + \frac{1}{RC} = 0$

$\Rightarrow \alpha = -\frac{1}{RC}$

At $t = 0 \Rightarrow q = Q_0 \Rightarrow b_2 = Q_0$.

4. $-\frac{1}{\alpha}$; Represents the time to reach 37% of Q_0, Or : $-\frac{1}{\alpha} = \tau$.

5. Decreasing exponentiel 0 ; Q_0 ; τ

III-

1. $i = \frac{dq'}{dt}$; $i = \frac{dq'}{dt} = -\frac{dq}{dt} \Rightarrow q'+q = \text{constant} = Q_0$.
2. $u_{MN} = \frac{q'}{C_2} + Ri = \frac{Q_0 - q}{C_2} - R\frac{dq}{dt}$.

3. $u_{MN} = \frac{Q_0 - q}{C_2} - R\frac{dq}{dt} = \frac{q}{C_1} \Rightarrow R\frac{dq}{dt} + q\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = \frac{Q_0}{C_2}$.
4. $\frac{dq}{dt} = \beta b_3 e^{\beta t} \Rightarrow R\beta b_3 e^{\beta t} + (a_3 + b_3 e^{\beta t})\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = \frac{Q_0}{C_2}$

$a_3 = \frac{Q_0 C_1}{C_1 + C_2}$; $b_3 = \frac{Q_0 C_2}{C_1 + C_2}$ and $\beta = -\frac{1}{R}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$

5. Decreasing exponential ; $a_3 = \frac{Q_0 C_1}{C_1 + C_2}$ $(t \to \infty \Rightarrow q = a_3)$; Q_0 , τ