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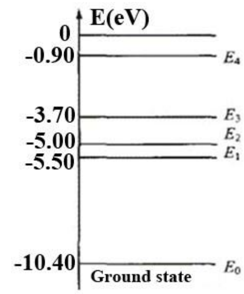
FACULTE DE GENIE

الجامعة اللبنانية
كلية الهندسة

Entrance exam 2014 – 2015 **PHYSICS** **6 July 2014**
Duration 2 H

Exercise I [18 pts]: Mercury atom



Given: mass of a mercury atom: $m_{Hg} = 3.34\times10^{-25}$ kg; mass of an electron: $m_e = 9.1\times10^{-31}$ kg;
$c = 3.00\times10^{8}$ m/s; $h = 6.63\times10^{-34}$ Js; 1 eV = 1.60×10^{-19} J.

A- Emission and absorption of a photon

1. One of the visible radiations emitted by a mercury vapor lamp corresponds to the transition from the energy level E_4 to the energy level E_3. Calculate the value of the corresponding wavelength $\lambda_{4/3}$.
2. Determine the value of the wavelength of the radiation that can be emitted by a mercury atom when it is taken initially in the energy level E_1.
3. A mercury atom is considered initially in the ground state E_0. This atom receives two photons of wavelengths $\lambda_1 = 253.7$ nm and $\lambda_2 = 589.0$ nm. Is there any interaction between the mercury atom and each of these two photons? Justify the answer.

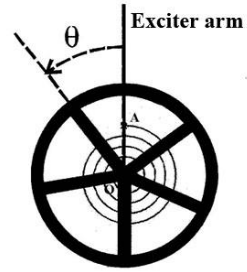
B- Collision between an electron and a mercury atom

In 1914, Franck and Hertz (Nobel Prize 1925) made a surprising discovery by bombarding a mercury vapor, **the atoms being supposed at rest**, with electrons of adjustable kinetic energy KE of a few eV.
1. We consider the case where KE is less than a certain threshold, $E_S = 4.90$ eV, and we suppose that the collision is perfectly elastic.
a) Show that the speed v_a of a mercury atom, after the collision, is given by $v_a = \frac{2\,m_e}{m_e + m_{Hg}} v$; v is the speed of the electron just before the collision, the velocities being collinear.
b) Deduce that the electron, after the collision, keeps practically the same kinetic energy KE.
2. a) When KE reaches the value KE = $E_S = 4.90$ eV, the electron, after the collision, loses practically all of its kinetic energy. Interpret this result.
b) For $E_S = 4.90$ eV < KE < 5.40 eV, the kinetic energy of some electrons, after the collision, diminishes precisely by 4.90 eV of its initial value, while the other electrons keep their kinetic energy KE. Interpret this result.
c) What could happen to mercury atoms that undergo collision with electrons having the kinetic energy KE = 6.00 eV?

C- Photoelectric Effect

When a potassium photocathode receives successively two radiations emitted by the mercury vapor lamp, one of wavelength $\lambda_1 = 253.7$ nm and the other $\lambda_2 = 444.0$ nm, we notice that the maximum kinetic energy of the ejected electrons are respectively 2.70 eV and 0.60 eV.
1. a) Using Einstein's relation and these data, determine the value of the Planck's constant h.
b) Deduce the work function W_S of the potassium photocathode.
2. Can the radiation due to the electronic transition $E_3 \rightarrow E_2$ contribute to the emission of a photoelectron? Why?

<u>Exercise II</u> [20 pts]: Free and forced oscillations (Pohl's pendulum)



The Pohl’s pendulum consists of a spiral spring (R) and of a copper wheel (D) that can rotate around a fixed horizontal axis (Δ) passing through its center O, J being its moment of inertia with respect to (Δ); (R), of elastic constant C, is connected from one side to (D) at O, and, from the other, to the exciter arm at A. (D) is shifted by an angle θ_0 ($\theta_0 > 0$) from its equilibrium position and then released without speed at the instant $t_0 = 0$. (D) starts to oscillate. At an instant t, the angular elongation of (D) is θ; its angular velocity is $\dot{\theta} = \frac{d\theta}{dt}$ and (R) exerts on (D) a couple of restoring torque $\Gamma = -C\theta$ and it thus stores an elastic potential energy $PE_e = ½ C\theta^2$, θ being in rad.

A-The exciter arm is fixed

1. a) Applying the conservation of the mechanical energy of the system (pendulum-Earth), derive the differential equation of motion of the pendulum in the absence of any damping force.

b) The solution of this differential equation is of the form $\theta(t) = \theta_m \cos(\omega_0 t + \varphi)$. Determine the expressions of the constants ω_0 and θ_m and the value of φ.

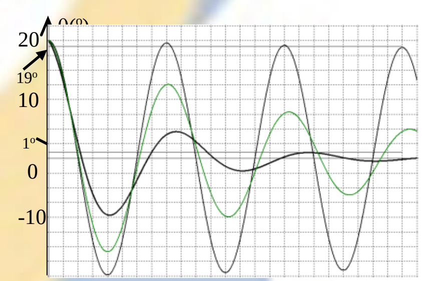
2. Assuming that (D) is subjected, at an instant t, in addition to the couple of torque Γ, to a braking couple of torque $M_f = -k\dot{\theta}$, where $k = k_0 + \beta I^2$; k_0 and β are constants, where k is adjusted by a current I carried by a system producing a magnetic field acting on the wheel (D) which will thus carry an induced current (called "Eddy current").

a) State the law interpreting the damping of (D) due to this induced current.

b) Show that the differential equation of motion of (D) can be written as: $\ddot{\theta} + 2\lambda\dot{\theta} + \omega_0^2\theta = 0$, where $\lambda = k/2J$ and $\omega_0 = \sqrt{C/J}$.

c) The solution of this differential equation is of the form: $\theta = B\, e^{-\lambda t} \cos(\omega t + \Phi)$, B and Φ being constants and ω the pseudo-angular frequency of expression: $\omega = \sqrt{\omega_0^2 - \lambda^2}$. Deduce the expression of the pseudo-period T.

d) Take $\delta = \ln\left(\frac{\theta(t)}{\theta(t+T)}\right)$. Show that $\delta = \lambda \cdot T$.



3. For different values of I ($I_1 = 0$ A, $I_2 = 0.400$ A then $I_3 = 0.700$ A), we record the three curves (a) for I_1, (b) for I_2 and (c) for I_3. (See adjacent figure)

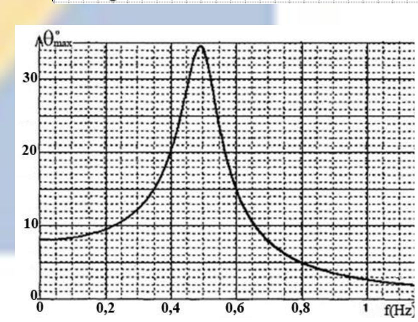
a) Determine, using the curve associated to I_2, the corresponding values of δ_2 and λ_2.

b) The curve giving the variations of λ as a function of I^2 is carried by a straight line.

i) Determine the equation giving λ as a function of I, knowing that for $I_1 = 0$ A, we have $\delta_1 = 0.017$ and $\lambda_1 = 0.0085\ s^{-1}$ and for $I_3 = 0.700$ A, we have $\delta_3 = 1.40$ and $\lambda_3 = 0.69\ s^{-1}$.

ii) Deduce the values of k_0 and β, knowing that $J = 10^{-4}\ kg \cdot m^2$.

c) Starting from what value of I do we obtain an aperiodic mode?



B-The exciter arm is made to move

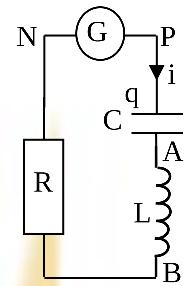
The exciter arm is made to move in an alternating sinusoidal motion of adjustable frequency f. By increasing f from zero, and by measuring the amplitude θ_m of the oscillations for each value of f, we obtain the graph giving the variations of θ_m as a function of f.

1. What phenomenon would be obtained for f close to 0.5 Hz? Interpret the answer.

2. How would the maximum amplitude θ_m(max) of oscillations vary when we increase I?

Exercise III [22 pts] : Maximum electric quantities and phase shift

A- Theoretical study



Consider the circuit of the adjacent figure. The coil is of inductance $L = 0.16$ H and of negligible resistance; the capacitor is of capacitance $C = 1.0\ \mu F$ and the resistor is of adjustable resistance R. The generator (G) maintains across its terminals an alternating sinusoidal voltage of adjustable angular frequency ω and of expression $u = u_{PN} = U_m \sin(\omega t + \varphi)$, where $U_m = 8.0$ V and $-\pi/2$ rad $< \varphi < \pi/2$ rad.

In steady state, the circuit carries, at an instant t, a current i of expression $i = I_m \sin(\omega t)$ and thus the capacitor carries a charge q. Take $LC\omega_0^2 = 1$.

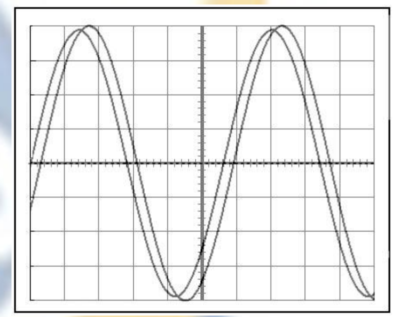
1. Determine, as a function of t and the data, the literal expressions of the voltages $u_R = u_{BN}$, $u_C = u_{PA}$ and $u_L = u_{AB}$.

2. a) Show, by applying the law of addition of voltages and giving t two specific values, that : $\tan\varphi = \dfrac{L\omega - \frac{1}{C\omega}}{R}$ and $I_m = \dfrac{U_m}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$.

b) Deduce the literal expressions of the amplitudes of U_{Cm} and U_{Lm} of u_C and u_L respectively.

B- Maximum value of U_{Cm} and phase shift

A voltmeter, in AC mode, is connected across the capacitor and an oscilloscope, suitably connected, displays the voltages u and u_R. Take $R = 250\ \Omega$. By increasing ω, starting from low values, we notice that U_{Cm} takes a maximum value $U_{Cm}(\max)$ for the value ω_C of ω. In this case, the adjacent figure shows the waveforms of the voltages u and u_R.



$S_V = 2$ V/div; $S_h = 0.5$ ms/div

1. Determine the value of ω_C.

2. Calculate the value of I_m and deduce that of U_{Cm} (max).

3. Determine the value φ_C of φ.

C- Maximum value of U_{Lm} and phase shift

The voltmeter, in AC mode, is now connected across the coil. Take $R = 250\ \Omega$. By increasing ω, starting from low values, we notice that U_{Lm} takes a maximum value $U_{Lm}(\max)$ for the value ω_L of ω.

1. Show that $\omega_L = \dfrac{\omega_0}{\sqrt{1 - \frac{R^2 C^2 \omega_0^2}{2}}}$ and calculate its value.

2. Calculate the value φ_L of φ and that of $U_{Lm}(\max)$.

D- Compare φ_L and φ_C, and also $U_{Cm}(\max)$ and $U_{Lm}(\max)$ and determine the expression among ω_L, ω_C and ω_0.

E- Maximum average power consumed by the circuit and phase shift

Take $\omega = 500\ \pi$ rad/s and R is of adjustable value.

1. Give, as a function of R and of the data, the literal expression of the average power P consumed by the circuit.
2. R is made to vary; we notice that the power P takes a maximum value P_1 for a value R_1 of R.
   a) Determine the value of R_1 and that of P_1.
   b) Calculate, in this case, the value φ_R of φ.

Solution of Entrance exam physics 2014-2015 **Exercise I: Mercury atom**

A-1	The energy of the emitted photon: $\Delta E = E_4 - E_3 = -0.90 - (-3.70) = 2.80$ eV $\Delta E = 2.80\times1.60\times10^{-19} = 4.48\times10^{-19}$ J. $\lambda_{4\to3} = \frac{hc}{\Delta E} = \frac{6.63\times10^{-34}\times3\times10^{8}}{4.48\times10^{-19}} = 4.440\times10^{-7}$ m or 444.0 nm (**violet-indigo**).	2
A-2	Energy of the emitted photon: $\Delta E = E_1 - E_0$; $\Delta E = -5.50 - (-10.40) = 4.90$ eV $\Rightarrow \Delta E = 4.90\times1.60\times10^{-19} = 7.84\times10^{-19}$ J. $\lambda_{1\to0} = \frac{hc}{\Delta E} = \frac{6.63\times10^{-34}\times3\times10^{8}}{7.84\times10^{-19}} = 2.537\times10^{-7}$ m = 253.7 nm, value less than 400 nm, (**UV – range**)	2
A-3	For the wavelength $\lambda_1 = 253.7$ nm, it is emitted during the transition $E_1 \to E_0$, so the atom taken initially in the state E_0 absorbs the photon and passes to the energy level E_1. **OR:** The energy of the photon received: $E = \frac{hc}{\lambda_1} = \frac{6.63\times10^{-34}\times3\times10^{8}}{253.7\times10^{-9}} \Rightarrow E = 7.84\times10^{-19}$ J $= \frac{7.84\times10^{-19}}{1.60\times10^{-19}} = 4.9$ eV. Let E_n be the energy level reached: $\Delta E_1 = E_n - E_0 = E_n - (-10.4) = 4.90$ eV $\Rightarrow E_n = -10.4 + 4.9 = -5.5$ eV. Thus the transition takes place from the level E_0 to the level E_1. Since λ_2 is greater than λ_1, then $\Delta E_2 < \Delta E_1$, thus the atom does not undergo any transition, so it does not interact	1.5
B-1-a	The system (electron - mercury atom) is pseudo-isolated, thus we have the conservation of the linear momentum of the system: $\vec{P}_{av} = \vec{P}_{ap} \Rightarrow m_e\vec{v} + \vec{0} = m_e\vec{v}' + m_{Hg}\vec{v}_a$; The algebraic measurement in the direction of the motion gives: $m_e v = m_e v' + m_{Hg} v_a$. $\Rightarrow m_e (v - v') = m_{Hg} v_a$. $\Rightarrow v - v' = \frac{m_{Hg}}{m_e} v_a$. (1) The collision being elastic, thus there is conservation of the kinetic energy of the system: $\frac{1}{2} m_e v^2 + 0 = \frac{1}{2} m_e v'^2 + \frac{1}{2} m_{Hg} v_a^2 \Rightarrow m_e (v^2 - v'^2) = m_{Hg} v_a^2$. (2) ; (2)/(1) $\Rightarrow v + v' = v_a$. (3) (3) and (1) $\Rightarrow 2v = (1 + \frac{m_{Hg}}{m_e}) v_a \Rightarrow v_a = \frac{2 m_e}{m_e + m_{Hg}} v$.	2.5
B-1-b	$v_a = \frac{2\times9.1\times10^{-31}}{9.1\times10^{-31} + 3.34\times10^{-25}} v \approx 5.5\times10^{-6}$ v. $\Rightarrow \frac{KE_{atom}}{KE_{electron}} = \frac{1/2\, m_{Hg} v_a^2}{1/2\, m_e v_e^2} \approx 10^{-5}$. The kinetic energy of the mercury atom, after the collision, is negligible with respect to that of the electron before the collision. According to the conservation of the kinetic energy, the electrons which have undergone collision keep substantially the same kinetic energy.	2
B-2-a	When KE reaches the value KE = E_S = 4.90 eV, an electron, after the collision, loses practically all of its kinetic energy. In fact, during this collision, the mercury atom taken initially in the ground state absorbs the kinetic energy that is exactly equal to the difference $E_1 - E_0 = 4.90$ eV ; the electron loses all of its kinetic energy.	1
B-2-b	For 4.90 eV < KE < 5.40 eV, the mercury atom, taken initially in the ground state, absorbs due to the collision, the energy 4.90 eV and passes to the energy level E_1 and the electron continues with a kinetic energy equal to KE – 4.90. The atom cannot pass to the energy level E_2 because $E_2 - E_0 = 5.40$ eV > KE. Electrons which do not interact with the atoms keep their kinetic energy KE.	1.5
B-2-c	For KE = 6.00 eV, the mercury atom initially taken in the ground state can pass after the collision, to the energy level $E_1 = -5.50$ eV or to the energy level $E_2 = -5.00$ eV ; since $KE - E_0 = 6.00 - 10.40 = -4.40$ eV > –5.00 eV > = – 5.50 eV. Thus, after the collision, a mercury atom can pass to the level E_1 by absorbing an energy of 4.90 eV and or to the level $E_2 = -5.00$ eV by absorbing an energy of 5.40 eV.	1.5
C-1-a	From the Einstein's relation: E(photon) = $\frac{hc}{\lambda} = W_S + KE(max)$ For $\lambda_1 = 253.7$ nm $\Rightarrow KE_1(max) = 2.70\times1.60\times10^{-19} = 4.32\times10^{-19}$ J For $\lambda_2 = 444.0$ nm $\Rightarrow KE_2(max) = 0.60\times1.60\times10^{-19} = 0.96\times10^{-19}$ J Thus : $KE_1(max) - KE_2(max) = hc[\frac{1}{\lambda_1} - \frac{1}{\lambda 2}] = hc\frac{\lambda_2 - \lambda_1}{\lambda_1\lambda_2}$ $\Rightarrow hc = \frac{[(4.32\times10^{-19} - 0.96\times10^{-19})\times253.7\times10^{-9}\times444\times10^{-9}]}{(444\times10^{-9} - 253.7\times10^{-9})} \Rightarrow h = 6.63\times10^{-34}$ J·s.	2.5
C-1-b	For $\lambda_1 = 253.7$ nm : E(photon) = 4.90 eV $\Rightarrow W_S = 4.90 - 2.70 = 2.20$ eV.	0.5
C-2	The energy of the photon due to the electronic transition $E_3 \to E_2$ is : $E_{3\to2} = -3.70 - (-5.00)$ $E_{3\to2} = 1.30$ eV < W_S, so there will be no emission of photoelectrons.	1

Exercise II: Free and forced oscillations (Pohl's Pendulum)

A-1-a	Mechanical energy of the system (pendulum, Earth): ME = KE + PE_e = $\frac{1}{2}J\theta'^2 + \frac{1}{2}C\theta^2$ = constant. The derivative of the ME with respect to time, gives: $\frac{1}{2}\,2\,J\theta'\theta'' + \frac{1}{2}\,2\,C\theta'\theta = 0$, with $\theta' \neq 0$, The differential equation of motion of the pendulum: $\theta'' + \frac{C}{J}\theta = 0$.	**2.5**
A-1-b	The solution of this differential equation: $\theta = \theta_m \cos(\omega_0 t + \varphi)$; $\theta' = -\omega_0\theta_m \sin(\omega_0 t + \varphi)$ and $\theta'' = -\omega_0^2\,\theta_m \cos(\omega_0 t + \varphi)$. Substituting in the differential equation, we get: $-\omega_0^2\,\theta_m \cos(\omega_0 t + \varphi) + + \frac{C}{J}\theta_m \cos(\omega_0 t + \varphi) = 0 \Rightarrow \omega_0^2 = \frac{C}{J}$ and $\omega_0 = \sqrt{\frac{C}{J}}$ At $t_0 = 0$, $\theta'_0 = -\omega_0\theta_m \sin(\varphi) = 0 \Rightarrow \varphi = 0$ or π. Similarly , at $t_0 = 0$, $\theta_0 = \theta_m \cos(\varphi) > 0 \Rightarrow \varphi = 0$ and $\theta_m = \theta_0$.	**3**
A-2-a	According to Lenz law, statement	**1.5**
A-2-b	According to the theorem of angular momentum: $\sum m_O = \frac{d\sigma}{dt} = J\theta''$ $\Rightarrow -C\theta - k\frac{d\theta}{dt} = J\theta''$ (the moments of the weight and of the reaction of the rotational axis with respect to (Δ) are zero) $\Rightarrow \theta'' + \frac{k}{J}\frac{d\theta}{dt} + \frac{C}{J}\theta = 0$. Thus, taking $\lambda = \frac{k}{2J}$ and $\omega_0^2 = \frac{C}{J}$, The differential equation can be written as: $\theta'' + 2\lambda\theta' + \omega_0^2\theta = 0$.	**2.5**
A-2-c	The pseudo-period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\omega_0^2 - \lambda^2}}$	**0.5**
A-2-d	As $\delta = \ln\frac{\theta_m e^{-\lambda t}\cos(\omega t + \varphi)}{\theta_m e^{-\lambda(t+T)}\cos[\omega(t+T)+\varphi]} \Rightarrow \delta = \ln\frac{1}{e^{-\lambda T}} \Rightarrow \delta = \lambda T$	**1.5**
A-3-a	For $I_2 = 0.4$ A, we find that $T_2 \approx 2$ s and that $\delta_2 = \ln\frac{20}{12.5} = 0.47$ $\Rightarrow \lambda_2 = \frac{\delta_2}{T_2} = \frac{0.47}{2} = 0.235\ s^{-1}$.	**2**
A-3-b-i	The graph of λ as a function of I^2 is carried by a straight line $\Rightarrow \lambda = a\,I^2 + b$. $0.69 = a\times0.7^2 + b$ and $0.235 = a\times0.4^2 + b$ $\Rightarrow a = 1.38\ A^{-2}$ and $b = 0.014\ s^{-1}$. So: $\lambda = 1.38\times I^2 + 0.014$ (I in A).	**2**
A-3 b-ii	$\lambda = \frac{k}{2J} \Rightarrow k = 2J\lambda \Rightarrow k = 2\times10^{-4}\times14\times10^{-3} + 2.76\times10^{-4}\,I^2$ $\Rightarrow k = 2.8\times10^{-6} + 2.76\times10^{-4}\,I^2$ $\Rightarrow k_0 = 2.8\times10^{-6}$ N·m·s/rad ; $\beta = 2.76\times10^{-4}$ N·m·s/rad·A^2.	**1.5**
A-3-c	For $\omega_0 = \lambda$, ω no longer exists, then there will be no oscillations So $\omega_0 = \lambda = 1.38\,I^2 + 0.014 = \frac{2\pi}{T_0} = \pi$ $\Rightarrow I^2 = \frac{(\pi - 0.014)}{1.38} = 2.27 \Rightarrow I = 1.51$ A.	**1**
B-1	For f close to 0.5 Hz we obtain the amplitude resonance phenomenon, because, when f is made to vary, the amplitude reaches a maximum for a value close to 0.5 Hz.	**1**
B-2	The maximum amplitude of oscillation decreases as I increases, since the damping increases with I.	**1**

Exercise III: Maximum electric quantities and phase shift

A-1	The voltage $u_{BN} = u_R$ across the resistor is: $u_{BN} = u_R = R\,i = R\,I_m\sin(\omega t)$ We have $i = \frac{dq}{dt}$ and $q = C\,u_{PA} = C\,u_C \Rightarrow i = C\frac{du_C}{dt}$. thus: $u_C = \frac{1}{C}\int i\,dt = -\frac{I_m}{C\omega}\cos(\omega t) + cte$ where the cte is zero because u_C is an alternating sinusoidal function $\Rightarrow$: $u_C = \frac{I_m}{C\omega}\cos(\omega t+\pi) = \frac{I_m}{C\omega}\sin(\omega t-\frac{\pi}{2})$. $u_{AB} = u_L = L\frac{di}{dt} = L\omega I_m\cos(\omega t) = L\omega I_m\sin(\omega t+\frac{\pi}{2})$.	**3**
A-2a	According to the law of addition of voltages: $u_{PN} = u_{PA} + u_{AB} + u_{BN}$. $U_m\sin(\omega t + \varphi) = \frac{I_m}{C\omega}\sin(\omega t - \frac{\pi}{2}) + L\omega I_m\sin(\omega t + \frac{\pi}{2}) + R\,I_m\sin(\omega t)$. For $\omega t = 0 \Rightarrow U_m\sin\varphi = -\frac{I_m}{C\omega} + L\omega I_m + 0 \Rightarrow U_m\sin\varphi = (L\omega - \frac{1}{C\omega})I_m$. (1) For $\omega t = \frac{\pi}{2} \Rightarrow U_m\cos\varphi = 0 + 0 + R\,I_m \Rightarrow U_m\cos\varphi = RI_m$. (2) (1)/(2) $\Rightarrow \tan\varphi = \frac{L\omega - \frac{1}{C\omega}}{R}$; $(1)^2 + (2)^2 \Rightarrow U_m^2 = I_m[R^2 + (L\omega - \frac{1}{C\omega})^2] \Rightarrow I_m = \frac{U_m}{\sqrt{R^2+\left(L\omega-\frac{1}{C\omega}\right)^2}}$	**3**
A-2b	The expressions of the amplitudes of u_C and u_L are respectively: $U_{Cm} = \frac{I_m}{C\omega} = \frac{U_m}{C\omega\sqrt{R^2+\left(L\omega-\frac{1}{C\omega}\right)^2}}$ and $U_{Lm} = L\omega I_m = \frac{L\omega U_m}{\sqrt{R^2+\left(L\omega-\frac{1}{C\omega}\right)^2}}$.	**1.5**
B-1	The period $T_C = S_h\times x = 0.5\times5.6 = 2.8$ ms ; the value of $\omega_C = \frac{2\pi}{T_C} = \frac{2\pi}{2.8\times10^{-3}} = 2244$ rad/s.	**1.5**
B-2	The value of $U_{Rm} = S_V\times y = 2\times3.9 = 7.8$ V. Thus, the value $I_m = \frac{U_{Rm}}{R} = \frac{7.8}{250} = 3.12\times10^{-2}$ A. The value of $U_{Cm}(max) = \frac{I_m}{C\omega_C} = \frac{3.12\times10^{-2}}{2244\times10^{-6}} = 13.9$ V	**2**
B-3	$\varphi_C = -2\pi\frac{d}{D} = -2\pi\frac{0.3}{5.6} = -0.336$ rad.	**1.5**
C-1	$U_{Lm} = L\omega I_m = \frac{L\omega U_m}{\sqrt{R^2+\left(L\omega-\frac{1}{C\omega}\right)^2}}$; U_{Lm} is maximum when its derivative with respect to ω is zero. $\Rightarrow \frac{dU_{Lm}}{d\omega} = \frac{L\,U_m\sqrt{R^2+\left(L\omega-\frac{1}{C\omega}\right)^2} - L\omega\,U_m\frac{2\left(L\omega-\frac{1}{C\omega}\right)\left(L+\frac{1}{C\omega^2}\right)}{2\sqrt{R^2+\left(L\omega-\frac{1}{C\omega}\right)^2}}}{[R^2+\left(L\omega-\frac{1}{C\omega}\right)^2]^2}$; $\frac{dU_{Lm}}{d\omega} = 0 \Rightarrow \sqrt{R^2+\left(L\omega_L-\frac{1}{C\omega_L}\right)^2} - \omega\frac{2\left(L\omega_L-\frac{1}{C\omega_L}\right)\left(L+\frac{1}{C\omega_L^2}\right)}{2\sqrt{R^2+\left(L\omega_L-\frac{1}{C\omega_L}\right)^2}} = 0$ $\Rightarrow R^2 + L^2\omega_L^2 + \frac{1}{C^2\omega_L^2} - 2\frac{L}{C} - L^2\omega_L^2 + \frac{1}{C^2\omega_L^2} = 0$ $\Rightarrow R^2 + \frac{2}{C^2\omega_L^2} - 2\frac{L}{C} = 0 \Rightarrow R^2 + \frac{2}{C^2\omega_L^2} - 2\frac{LC}{C^2} = 0 \Rightarrow R^2 + \frac{2}{C^2\omega_L^2} - 2\frac{1}{C^2\omega_0^2} = 0$ $\Rightarrow \frac{R^2C^2}{2} = \frac{1}{\omega_0^2} - \frac{1}{\omega_L^2} \Leftrightarrow \frac{1}{\omega_L^2} = \frac{1}{\omega_0^2} - \frac{R^2C^2}{2} = \frac{1}{\omega_0^2}[1 - \frac{R^2C^2\omega_0^2}{2}] \Rightarrow \omega_L = \frac{\omega_0}{\sqrt{1-\frac{R^2C^2\omega_0^2}{2}}} = 2787$ rad/s.	**2.5**
C-2	We have $\tan\varphi_L = \frac{L\omega_L - \frac{1}{C\omega_L}}{R} = \frac{0.16\times2787 - \frac{1}{10^{-6}\times2787}}{250} = 0.348 \Rightarrow \varphi_L = +0.335$ rad. $U_{Lm}(max) = \frac{L\omega_L U_m}{\sqrt{R^2+\left(L\omega_L-\frac{1}{C\omega_L}\right)^2}} = \frac{0.16\times2787\times8}{\sqrt{250^2+\left(0.16\times2787-\frac{1}{10^{-6}\times2787}\right)^2}} = 13.47$ V	**1.5**
D	We find that $\varphi_L = -\varphi_C$, $U_{Cm}(max) = U_{Lm}(max)$ and that $\omega_L\times\omega_C = 2787\times2244 = 6254000 \approx \omega_0^2 = 2500^2 = 6250000$ rad^2/s^2	**1**
E-1	The expression of the average power P consumed by the circuit is given by: $P = RI^2 = \frac{1}{2}R\,I_m^2 = \frac{RU_m^2}{2[R^2+\left(L\omega-\frac{1}{C\omega}\right)^2]}$.	**1**
E-2a	The average power P is maximum when $\frac{dP}{dR} = 0$. $\frac{dP}{dR} = \frac{1}{2}\frac{U_m^2[R^2+\left(L\omega-\frac{1}{C\omega}\right)^2] - RU_m^2\cdot2R}{[R^2+\left(L\omega-\frac{1}{C\omega}\right)^2]^2}$; for $R = R_1$; $\frac{dP}{dR} = \frac{1}{2}\frac{U_m^2[\left(L\omega-\frac{1}{C\omega}\right)^2 - R_1^2]}{[R_1^2+\left(L\omega-\frac{1}{C\omega}\right)^2]^2} = 0$ $\Rightarrow \left(L\omega - \frac{1}{C\omega}\right)^2 - R_1^2 = 0 \Rightarrow R_1 = \left\lvert L\omega - \frac{1}{C\omega}\right\rvert$, since $R_1 > 0$. $R_1 = \left\lvert 0.16\times500\pi - \frac{1}{10^{-6}\times500\pi}\right\rvert = 385\ \Omega$. $P_1 = \frac{R_1U_m^2}{2[R_1^2+R_1^2]} = \frac{R_1U_m^2}{4R_1^2} = \frac{U_m^2}{4R_1} = \frac{8^2}{4\cdot385} = 4.15\times10^{-2}$ W.	**2.5**
E-2b	In this case, $\tan\varphi_R = \frac{L\omega - \frac{1}{C\omega}}{R_1} = -1 \Rightarrow \varphi_R = -\frac{\pi}{4}$ rad.	**1**