LEBANESE UNIVERSITY
FACULTY OF ENGINEERING

الجامعة اللبنانية
كلية الهندسة

Entrance Exam 2001-2002 **Physics** **Duration: 2 hours**

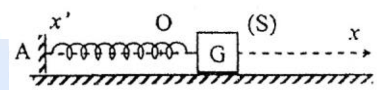
Remark: The use of a calculator with no programs is allowed.

First exercise.

A- Theoretical study

I- A horizontal mechanical oscillator is formed of a solid (S) of masse M = 0.760 kg fixed to a spring (R) of force constant K = 8.3 N/m and of negligible mass.

(S) can slide without friction on a horizontal axis x'x. When (S) is in equilibrium, its center of inertia G is situated at the point O considered as the origin of abscissa. (S) is shifted by X_0 = 3.7 cm in the positive direction, and then released from rest at the instant to = 0. .



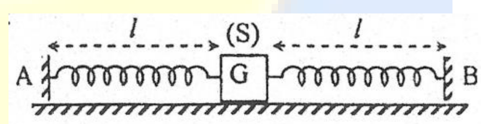
At the instant t, G being at a point of abscissa x has the velocity $\vec{V} = V\vec{i}$, ($V = \frac{dx}{dt} = \dot{x}$).

The horizontal plane containing x'x is chosen as the reference level for the gravitational potential energy.

1. Give, at the instant t, the expression of the mechanical energy of this oscillator in terms of x and $\dot{x}$, and deduce the differential equation that describes the motion of G.
2. Determine the values of the characteristic quantities ω_0 and T_0 of this oscillator.
3. Determine the time equation of the motion of G.

II- (S) is attached to two springs identical to (R) each of natural length ℓ_0. The length of the spring at equilibrium is ℓ ($\ell > \ell_0$). We consider the same preceding conditions (x_0 = 3.7 cm and V_0 = 0)

The two springs are always stretched and (S) oscillates without friction along AB.



1. Calculate, in terms of ℓ_0, ℓ and x the elongation of each spring at any instant t and give the expression of the elastic potential energy of each spring.
2. a) Write the mechanical energy of this new oscillator at any instant t, and then deduce the differential equation that describes the motion of G.

   b) Deduce that the value of the proper pied $T_0^{'}$ of this oscillator is $T_0^{'} = T_o/\sqrt{2}$.

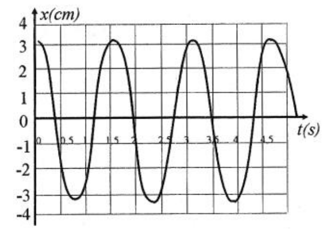
B- Experimental study:

I- Free oscillations.

We consider the oscillator where (S) is attached to the two springs identical to (R).

An appropriate apparatus allows to record the motion of the center of inertia G of (S) as a function of time. The recording of the different positions of G gives the adjacent curve.

1. Does the mechanical energy of this oscillator remain conserved during oscillations? Justify.
2. Measure, using this curve, a characteristic quantity of this oscillator and compare it to that calculated in the theoretical study.



II- Forced oscillations.

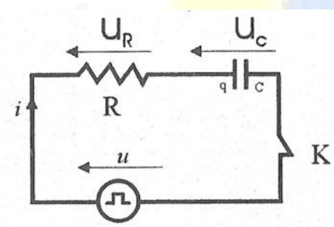
The oscillator is in its equilibrium position.

The end A is no more fixed; it is connected to an exciter of adjustable frequency.

A is then performing a sinusoidal motion of frequency f. The oscillator [(S), springs, support] is the resonator which performs oscillations of amplitude X_m depending on f. Give the shape of the curve representing the variations of X_m as a function of f specifying the value of the frequency at amplitude resonance.

Second exercise.

A circuit is formed of the components placed in series, a switch(K), a resistor (R) of resistance R and a capacitor (C) of capacitance C . This circuit is fed by a low frequency generator delivering across its terminals a square wave.



u. = U for $0 \leq t \leq T/2$;

u = 0 for $T/2 \leq t \leq T$.

(K) is closed at the instant $t_0 = 0$

1. a) Derive the differential equation that describes the evolution of u_C as a function of time t for $0 \leq t \leq T/2$.
   b) Give the expression of the time constant τ of this circuit.
   c) Give the shape of the curve u_C as a function of time supposing that $T/2 \gg \tau$ and specify the value of u_C at the instant T/2.
2. a) What will happen starting from the instant T/2?
   b) Write, then, the differential equation that describes the evolution of u_C as a function of time t for $T/2 \leq t \leq T$.
   c) Give the shape of the curve showing the evolution of u_C and that of u_R as a function of time t for $T/2 \leq t \leq T$.

Third exercise.

Comparison of nuclear reactions:

I. The mass of a α particle can be considered equal to 4.0039 u where u is the atomic mass unit.

1. Define the atomic mass unit.
2. When an α particle hits a beryllium nucleus $^{9}_{4}Be$, a nucleus is formed and a neutron is emitted.
   a) Write the equation of this nuclear reaction specifying the considered laws of conservation.
   b) Identify the obtained nucleus.

II- When a neutron hits a uranium nucleus $^{235}_{92}U$ a fission occurs. Among the possible nuclear reactions, we study the balance of the following fission:

$$^{1}_{0}n + ^{235}_{92}U \longrightarrow ^{139}_{54}Xe + ^{95}_{38}Sr + 2\,^{1}_{0}n$$

Given:

$m_n = 1.00866$ u , $m_p = 1.00728$ u , 1 u = $1.66055.10^{-27}$ kg = 931.5 MeV/c^2.

Binding energy per nucleon for: $^{235}_{92}U$: $E_A(U) = 7.7$ MeV

Binding energy per nucleon for $^{139}_{54}Xe$: $E_A(Xe) = 8.4$ MeV

Binding energy per nucleon for $^{95}_{38}Sr$: $E_A(Sr) = 8.7$ MeV.

A. 1. Give the expression of the mass of a nucleus in terms of c, the mass m_n of a neutron, the mass m_p of a proton and the binding energy E_B of this nucleus.

2. Calculate the respective masses m_1, m_2 and m_3 of the nuclei U, Xe and Sr.
3. Verify that this reaction is exoenergetic.

B. Knowing that the natural nuclides $^{132}_{54}Xe$ and $^{88}_{38}Sr$ are stable, the products of the fission are radioactive; they are β^- emitter.

1. These products transform into other radioactive elements. The whole set of these products constitutes radioactive waste. Among these waste, we find strontium ^{90}Sr of period (half-life) 25 years and the cesium ^{137}Cs of period 100/3 years.
   a) Define the radioactive period of a radio-nuclide.
   b) If N_0 is the number of ^{90}Sr nuclei at the instant $t_0 = 0$ and N that of the remaining ^{90}Sr nuclei at the instant $t_1 = 100$ years, determine the ratio N'/N_0 where N' is the number of the disintegrated ^{90}Sr nuclei during these 100 years.

2. After many β^- disintegrations, the products of the fission end to two stable nuclides: the lanthanum $_{57}La$ La and the molybdenum $_{42}Mo$.

a) Write the total balanced equation of these disintegrations specifying the mass number for each stable nuclide.

b) Deduce the balanced equation due to the fission of the $^{235}_{92}U$ nucleus that leads to the stable nuclides.

c) During these disintegrations, are there neutrinos or antineutrinos emitted? For what reason was the neutrino or the antineutrino introduced?

Entrance Exam 2001-2002 **Solution of Physics** **Duration: 2 hours**

First exercise:

A-1-a- ME = $\frac{1}{2}Kx'^2 + \frac{1}{2}K\,x^2$ or = $0.38\,x'^2 + 4.15\,x^2$

At t = 0 ; v = 0 ; x = 0.037m ⇔ ME = 0.0057J

Since (S) slides without friction ⇔ ME is conserved ……………⇔ x'' + 11x = 0.

b- Similar to $x'' + \omega_0^2\,x = 0$ ⇔ by comp … $\omega_0 = \sqrt{11}$ rds/s ⇔ $T_0 = 1.9$ s.

c- x = 0.037 cos3.3t (x in m)

2-a- at equilibrium $\Delta\ell = \ell - \ell_0$. … $PE_1 = \frac{1}{2}K(\ell - \ell_0 + x)^2$

…...$PE_2 = \frac{1}{2}K(\ell - \ell_0 - x)^2$

b- i - ME at any time ………………..

x'' + 22x = 0

ii - new differential equal.. $y'' + \omega'^2_0\,y = 0$ ……….. $\omega'_0 = \sqrt{2}\,\omega_0$ ⇔ $T'_0 = \frac{T_0}{\sqrt{2}}$.

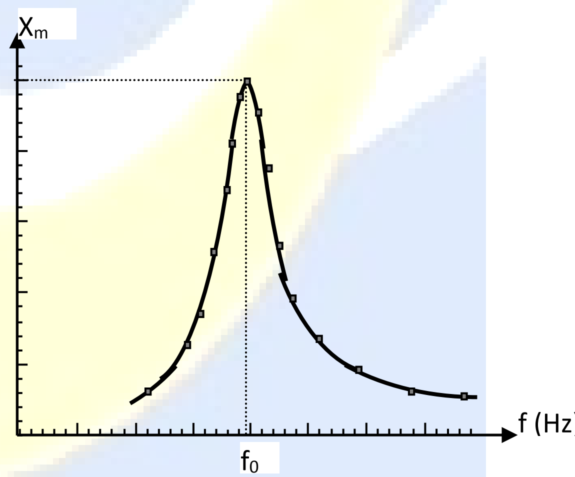
B-I- a- From the graph the amplitude is decreasing then the motion is pseudo-periodic.

b-The period is T = 1.35 sec from the graph

theoretical period is $T'_0 = \frac{T_0}{\sqrt{2}}$ = 1.35 sec

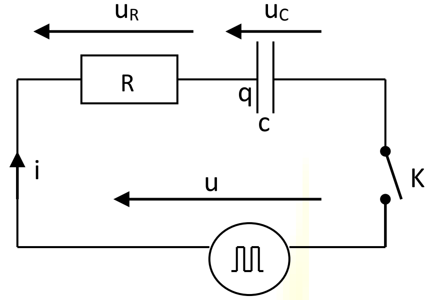
$T \approx T_0'$.

II- In the case of forced oscillations, the frequency of the resonator is the same as that of the exciter, but the amplitude varies with the frequency.
This amplitude is a maximum when the frequency of the exciter is equal to the proper frequency of the resonator.
The adjacent graph represents the variation of X_m as a function of f.



The resonance is sharp since we have slight damping. The value of f_0 is : $f_0 = \frac{1}{T_0'}$ = 0.74 Hz

Second Exercice



1- a- $u = u_C + u_R = u_C + Ri$………………..

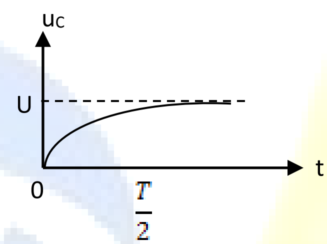
$u'_C + \frac{1}{RC} u_C = \frac{u}{RC}$.

For $0 \le t \le T/2 \Leftrightarrow u'_C + \frac{1}{RC} u_C = \frac{U}{RC}$.

b- $u_C = U(1 - e^{-\frac{t}{\tau}})$ where $\tau = RC$ is the time constant.

c- Since $\frac{T}{2} >> \tau$. Then for $t = \frac{T}{2}$ we get:

$e^{-\frac{t}{\tau}} \approx 0 \Leftrightarrow u_C = U$.

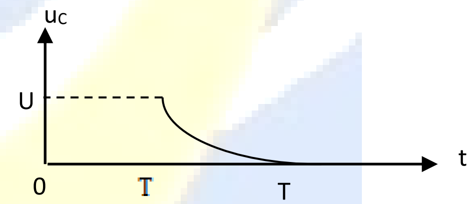


2- a- Starting from T/2 , we have u = 0 and the condenser is discharging in the resistor and u_C decreases to 0.

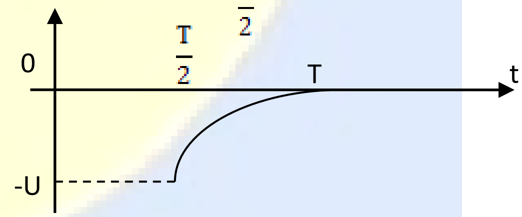
b- for $\frac{T}{2} \le t \le T$ we have u = 0, we replace in equation , $u'_C + \frac{1}{RC} u_C = \frac{U}{RC}$.

u by 0 we get $u'_C + \frac{1}{RC} u_C = 0$.

c- The solution is $u_C = U e^{-\frac{t}{\tau}}$, its curve is the following :



Since $u = u_C + u_R = 0 \Rightarrow u_R = -u_C$, then the curve of u_R is the following :



Third exercise:

1- a- The atomic mass unit is $\frac{1}{2}$ of the mass of ${}^{12}_{6}C$ nucleus (1 u = 1.66 × 10^{-27}Kg)

b- i – The equation of the reaction is : ${}^{4}_{2}He + {}^{9}_{4}Be \rightarrow {}^{1}_{0}n + {}^{A}_{Z}X$

The conservation laws give: - Mass number is conserved: A = 12

Atomic number is conserved: Z = 6

ii – Since ${}^{A}_{Z}X \Rightarrow$ X is a carbon ${}^{12}_{6}C$ nucleus.

2- a- i- Einstein's law implies : $E_\ell = \Delta mc^2$.

With Δm = mass of nucleons – mass of nucleus.

$\Delta m = Zm_P + (A- Z)m_n - m$

$\Rightarrow \frac{E_\ell}{c^2} = \Delta m =$ ………………. $\Rightarrow m = Zm_P + Nm_n - \frac{E_\ell}{c^2}$

ii- Given : $m_pc^2 = 1.00728 \times 931.5 = 938.28132$ MeV

and $m_nc^2 = 939.56679$ MeV

$\Rightarrow m_1c^2 = 92\times938.28132 + (235 - 92)\times939.56679 - 7.7 \approx 220672$ MeV $\Rightarrow m_1 \approx 220672$ Mev/c^2

Similarly $m_2 = 130522$ MeV/c^2.

$m_3 = 89201$ MeV/c^2.

iii- The difference between the mass of reactants and products is:

m (product) – m (reactant) = $m_2 + m_3 + 2m_n - (m_1 + m_n) = - 9.5$ MeV/c^2

Since Δm < 0 the reaction is exoergic.

b- 1- a – The period is the time needed for the activity tp be reduced to ½ its initial value

b – Since the period of Sr is 25 then t = nT , so n = 4 $\Rightarrow N = \frac{N_0}{2^4} = \frac{N_0}{16}$, the number of disintegrated nuclei is : $N' = N_0 - N = N_0 (\frac{15}{16})$

2- a – For Xe :

${}^{139}_{54}Xe \rightarrow {}^{A_1}_{57}La + x\ {}^{0}_{-1}e + x\ {}^{0}_{0}\bar{\nu}$; Such that A = 139 ; x = 3

$${}^{139}_{54}Xe \rightarrow {}^{139}_{57}La + 3\ {}^{0}_{-1}e + 3\ {}^{0}_{0}\bar{\nu}$$

Similar for Sr:

$${}^{95}_{38}\mathrm{Sr} \rightarrow {}^{A_2}_{42}\mathrm{Mo} + y\ {}^{0}_{-1}\mathrm{e} + y\ {}^{0}_{0}\bar{\nu}$$; $A_2 = 95$; $y = 4$.

b- The global fission reaction:

$${}^{1}_{0}\mathrm{n} + {}^{235}_{92}\mathrm{U} \rightarrow {}^{139}_{57}\mathrm{La} + {}^{95}_{42}\mathrm{Mo} + 2\ {}^{1}_{0}\mathrm{n} + 7\ {}^{0}_{-1}\mathrm{e} + 7\,{}^{0}_{0}\bar{\nu}$$

c- The particles introduced are antineutrino and are given to maintain energy conservation in β^- decay reactions.