LEBANESE UNIVERSITY

FACULTY OF ENGINEERING

الجامعة اللبنانية
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Entrance Exam 2017 - 2018 — ***Mathematics*** — ***Duration : 3 hours***

The distribution of grades is over 50 — ***July 08 , 2017***

Exercise 1 (7 points)

The complex plane is referred to a direct orthonormal system $(O;\ \vec{u}, \vec{v})$.

A- Let A be the point of affix 10 and (γ) the circle of diameter $[OA]$.

1- Prove that the points B and C of respective affixes $b = 1+3i$ and $c = 8-4i$ belong to (γ) .

2- Let D be the point of affix $d = 2+2i$.

Calculate $\dfrac{b-d}{b-c}$ and $\dfrac{d}{b-c}$. Deduce that D is the orthogonal projection of O on (BC).

Draw (γ) and plot the points A, B, C and D.

B- To each point M of plane with affix z, distinct from O, we associate the point M' of affix z' such that $z' = \dfrac{20}{\bar{z}}$.

1- Prove that the points O, M and M' are collinear .

2- Suppose in this part that M belongs to the straight line (Δ) of equation $x = 2$.

a) Verify that $z + \bar{z} = 4$ and prove that $5(z' + \overline{z'}) = z'\overline{z'}$. Deduce that M' belongs to (γ) .

b) Take a point M on (Δ) and plot the associated point M'.

Exercise 2 (7 points)

30% of the students of a high school are members of the " extracurricular activities club " (EAC).
We know that one quarter of girls and one third of boys of the school are members of the EAC .

A- A student is chosen randomly from the high school . Consider the two events :
G : " the chosen student is a girl " and A : " the chosen student is a member of the EAC " .

1- a) Prove that the probability of the event G is equal to $\dfrac{2}{5}$.

b) Calculate the probability that the chosen student is a boy not a member of the EAC .

2- We choose a student in the EAC . What is the probability that this student is a girl ?

B- To finance the school ceremony for the national day , the EAC organizes a lottery .
Each day , a student is randomly and independently chosen from the school to hold the lottery .

1- Determine the probability that, among the students chosen in a week of 5 days, there are exactly two members of the EAC .

2- For any non zero natural number n , denote by p_n the probability that in n consecutive weeks , there is at least one member of the EAC chosen . Prove that $p_n = 1 - \left(\dfrac{7}{10}\right)^{5n}$.

3- Determine the minimum number of weeks so that $p_n > 0.999$.

Exercise 3 (7 points)

1- Consider the functions f and h defined on the interval $K=[1;2]$ by :
$f(x)=1+2\ell n(x+1)-\ell n(x^2+1)$ and $h(x)=f(x)-x$.
a) Prove that the two functions f and h are strictly decreasing in K .
b) Prove that if $x\in K$, then $f(x)\in K$.
c) Prove that the equation $f(x)=x$ has a unique solution α .

2- Consider the sequence (U_n) of first term $U_0=\frac{1}{5}$ such that , for all natural numbers n , $U_{n+1}=f(U_n)$.

a) Prove that , for all $n\geq 1$, $1\leq U_n\leq 2$.

b) We admit that , for all $x\in K$, $|f'(x)|\leq\frac{1}{4}$.

Knowing that , for all $x\in K$, we have $|f(x)-\alpha|\leq\frac{1}{4}|x-\alpha|$, prove that for all $n\geq 1$,

$|U_{n+1}-\alpha|\leq\frac{1}{4}|U_n-\alpha|$.

c) Prove by induction that , for all natural numbers n , $|U_n-\alpha|\leq\left(\frac{1}{4}\right)^{n-1}$.

Deduce the limit of the sequence (U_n) .

Exercise 4 (9 points)

The plane is referred to a direct orthonormal system $(O;\vec{i},\vec{j})$.

Consider the ellipse (γ) of equation $\frac{x^2}{4}+(y+1)^2=1$.

1- Draw (γ) . (***Unit : 2 cm***)

2- Calculate the area of the domain interior to (γ) . Deduce $\int_0^2\sqrt{4-x^2}\,dx$.

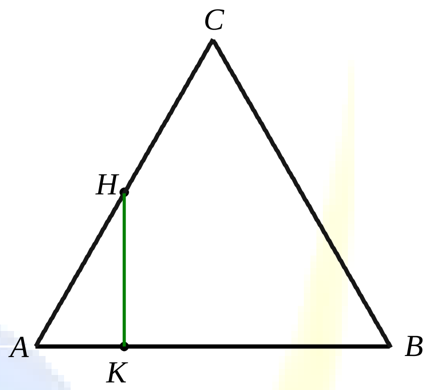
3- Let F_1 and F_2 be the foci of (γ) (F_1 is the focus having positive abscissa) , (d_1) the directrix associated to F_1 and $M(\alpha;\beta)$ where $\beta\neq -1$, a point on (γ) .
a) The tangent (δ) to the ellipse (γ) at M cuts (d_1) at L . Prove that the angle $L\hat{F_1}M$ is right .
b) Plot the point M on (γ) and describe a geometric construction of the tangent (δ) .

4- Let θ be the measure in radians of the angle $F_1\hat{M}F_2$.
a) Calculate MF_1 in terms of α and deduce MF_2 .
b) Prove that $\cos\theta=\frac{3\alpha^2-8}{16-3\alpha^2}$ and determine θ when M is one of the vertices of (γ) that belong to the non focal axis .
c) Determine the abscissas of the points of (γ) that are also on the circle of diameter $[F_1F_2]$.

Exercise 5 (9 points)

In an oriented plane , consider an equilateral triangle ABC such that $(\overrightarrow{AB}\,;\,\overrightarrow{AC}) = \frac{\pi}{3}\ (2\pi)$.

Let H be the mid point of $[AC]$ and K the orthogonal projection of H on $[AB]$.



1- Let S be the similitude of center A that transforms K into H and S' the similitude that transforms B into H and H into K.
Determine the ratio and an angle of each of the similitudes S and S'.

2- Consider the transformation $T = S \circ S'$.
a) Determine $T(H)$ and precise the nature and the elements of T .
b) Determine $T(C)$. Deduce that $S'(C) = A$.

3- Let I be the mid point of $[AB]$.
a) Justify that K is the mid point of $[AI]$. Deduce that $S'(A) = I$.
b) Determine the point $J = S'(I)$.

4- Consider the transformation $f = S' \circ S' \circ S'$.
a) Determine the nature and the elements of f .
b) Prove that $f(C) = J$. Deduce the center L of S'.

5- The plane is referred to the direct orthonormal system $(A\,;\vec{u}\,;\vec{v})$ where $\vec{u} = \frac{1}{4}\overrightarrow{AB}$.
a) Determine the complex relation of S'.
b) Deduce the affix of J and that of the center L of S'.

Exercise 6 (11 points)

The plane is referred to an orthonormal system $(O\,;\vec{i}\,,\vec{j})$.

A- Let T be the transformation that , to each point $M(x\,;y)$, associates the point $N(x'\,;y')$ such that $x' = -x$ and $y' = -2x + y$.

1- a) Prove that $\overrightarrow{MN}$ is collinear to $\vec{i} + \vec{j}$ and the mid point P of $[MN]$ belongs to the axis of ordinates .
b) Describe the geometric construction of the image N of any point M of the plane .

2- a) Prove that any point of the axis of ordinates is invariant by T .
b) Let (d) be a straight line of director coefficient a . Prove that the image of (d) by T is a straight line (d') and that the straight lines (d) and (d') intersect on the axis of ordinates .

B- 1- a) Set up the table of variations of the function g defined on $]-\infty\,;0]$ by $g(x)=x-1+2e^{x}$.

b) Set up the table of variations of the function h defined on $[0\,;+\infty[$ by $h(x)=x-1+2e^{-x}$.

2- Let (C_1) and (C_2) be the representative curves of g and h respectively (It is not required to draw them) .

a) Prove that (C_2) is the image of (C_1) by T .

b) Prove that the straight line (δ) of equation $y=x-1$ is asymptote to (C_1) and to (C_2) .

c) Determine the position of each of the curves (C_1) and (C_2) with respect to (δ) .

3- Consider the function f defined on IR by $f(x)=x-1+2e^{-|x|}$. Let (C) be its representative curve .

a) Prove that (C) is the union of (C_1) and (C_2) .

b) Precise the semi tangents to (C) at the point A of abscissa 0 .

c) Draw (C) (***Graph unit*** : **2 cm**) .

4- Let (Δ) be the straight line of equation $y=x-1+2m$ where $m\in]0\,;1[$.

a) Prove that , for all $m\in]0\,;1[$, (Δ) cuts (C) at two points : E on (C_1) and F on (C_2) .

b) Verify that $F=T(E)$ (T is the transformation defined in part ***A***) .

5- Let (t_1) be the tangent at E to (C_1) and (t_2) the tangent at F to (C_2) .

Knowing that (t_1) and (t_2) intersect on the axis of ordinates , deduce that (t_2) is the image of (t_1) by T .

Entrance Exam 2017 - 2018 **Mathematics (SOLUTION)** **July 08 , 2017**

(Program : Lebanese bac)

Exercise 1 (7 points)

A- (γ) is the circle of diameter $[OA]$ of center the point I with affix 5, the mid point of $[OA]$, and radius 5 .

1- $IB=|b-5|=|-4+3i|=5$ then , B belongs to (γ).

$IC=|c-5|=|3-4i|=5$ then C belongs to (γ).

2- D is the point of affix $d=2+2i$;

$\dfrac{b-d}{b-c}=\dfrac{-1+i}{-7+7i}=\dfrac{1}{7}$ and

$\dfrac{d}{b-c}=\dfrac{2+2i}{-7+7i}=\dfrac{2i(1-i)}{-7(1-i)}=-\dfrac{2}{7}i$.

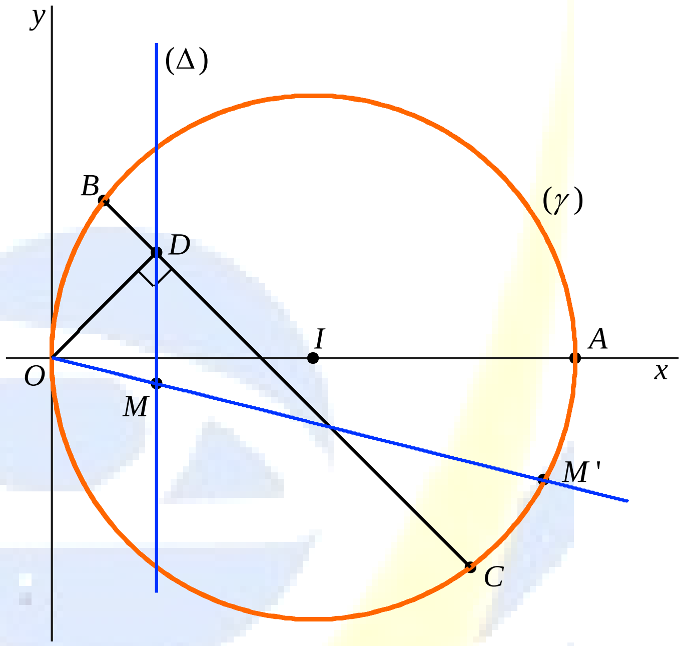
$\dfrac{b-d}{b-c}$ is real then , $D\in(BC)$;

$\dfrac{d}{b-c}$ is pure imaginary then , (OD) is perpendicular to (BC).

Therefore (OD) is perpendicular to (BC) at D ;

that is D is the orthogonal projection of O on (BC).

Drawing a figure showing (γ) , A, B and C .



B- 1- $\dfrac{z'}{z}=\dfrac{20}{z\,\bar{z}}$ then , $\dfrac{z'}{z}$ is a pure positive real number ; therefore $(\overrightarrow{OM}\ ;\ \overrightarrow{OM}')=0\ \ (2\pi)$ and the points O, M and M' are collinear .

2- (Δ) is the straight line of equation $x=2$ and M a point of (Δ) then , $z=2+yi$ with $y\in IR$.

a) $z+\bar{z}=2+yi+2-yi=4$.

$z'+\overline{z'}=\dfrac{20}{\bar{z}}+\dfrac{20}{z}=\dfrac{20(z+\bar{z})}{z\,\bar{z}}=\dfrac{80}{z\,\bar{z}}$; $5(z'+\overline{z'})=\dfrac{400}{z\,\bar{z}}=\dfrac{20}{\bar{z}}\times\dfrac{20}{z}=z'\,\overline{z'}$.

$IM'^2=(z'-5)(\overline{z'-5})=z'\,\overline{z'}-5(z'+\overline{z'})+25=25$ then , $IM'=5$ and $M'\in(\gamma)$.

Therefore , M' is the point of intersection of (OM) and (γ).

b) Plotting M' on the figure as the point where (OM) cuts (γ) .

Exercise 2 **(7 points)**

A- It is given that $p(A)=\dfrac{3}{10}$, $p(A/G)=\dfrac{1}{4}$ and $p(A/\overline{G})=\dfrac{1}{3}$.

1- a) Let $x=p(G)$. By the law of total probability ,

$P(A)=p(A\cap G)+p(A\cap\overline{G})=p(G)\times p(A/G)+p(\overline{G})\times p(A/\overline{G})$, then $\dfrac{3}{10}=x\times\dfrac{1}{4}+(1-x)\times\dfrac{1}{3}$.

Therefore , $\dfrac{1}{12}x=\dfrac{1}{30}$, then $x=\dfrac{2}{5}$; that is $p(G)=\dfrac{2}{5}$.

b) The required probability is $p(\overline{G}\cap\overline{A})=p(\overline{G})\times P(\overline{A}/\overline{G})=p(\overline{G})\times(1-P(A/\overline{G}))=\dfrac{3}{5}\times\dfrac{2}{3}=\dfrac{2}{5}$.

2- The required probability is $p(G/A)=\dfrac{p(G\cap A)}{p(A)}=\dfrac{p(G)\times p(A/G)}{p(A)}=\dfrac{2}{5}\times\dfrac{1}{4}\div\dfrac{3}{10}=\dfrac{1}{3}$.

B- 1- The 2 days of choosing a member of the EAC can be selected in ${}_5C_2=10$ ways ;

in addition $p(A)=\dfrac{3}{10}$ and $p(\overline{A})=\dfrac{7}{10}$; therefore , the required probability is

$$p=10\times\left(\frac{3}{10}\right)^2\times\left(\frac{7}{10}\right)^3=\frac{30870}{100000}=0.3087.$$

2- In n weeks there are $5n$ days . Consider the event :

E : " no student is a member of the EAC " ; $p(E)=\left(\dfrac{7}{10}\right)^{5n}$.

The required probability is $p_n=p(\overline{E})=1-\left(\dfrac{7}{10}\right)^{5n}$

3- We have to solve the inequality $1-\left(\dfrac{7}{10}\right)^{5n}>0.999$ which is equivalent to $\ell n\left((0.7)^{5n}\right)<\ell n(0.001)$;

that is $5n\,\ell n(0.7)<\ell n(0.001)$; $n>\dfrac{\ell n(0.001)}{5\,\ell n(0.7)}\approx 3.87$.

Therefore we need at least 4 weeks for having $p_n>0.999$.

Exercise 3 **(7 points)**

1- a) $f'(x)=\dfrac{2}{x+1}-\dfrac{2x}{x^2+1}=\dfrac{2(1-x)}{(x+1)(x^2+1)}$.

For all x in $]1\,;2[$, $f'(x)<0$ then f is strictly decreasing in K.

$h'(x)=f'(x)-1$ where $f'(x)\le 0$ then , for all x in K, $h'(x)<0$ and h is strictly decreasing in K.

b) f is continuous and strictly decreasing in K then , for all x in K, $f(2)<f(x)<f(1)$ where $f(1)=1+\ell n2<2$ and $f(2)=1+\ell n9-\ell n5>1$ then , $f(x)\in K$.

c) h is continuous and strictly decreasing in K then , $h(K)=[h(2)\,;h(1)]$ where $h(1)=\ell n2\approx 0.693$ and $h(2)=-1+\ell n9-\ell n5\approx -0.412$.

h is a bijection of K into the interval $h(K)$ that contains 0 then , the equation $h(x)=0$ which is equivalent to $f(x)=x$ has a unique solution α in K.

2- a) $U_1=f(U_0)=1+\ell n\dfrac{18}{13}\approx 1.325$ then , $U_1\in K$.

If , for a certain all $n\ge 1$, $U_n\in K$ then , $f(U_n)\in K$ (proved in 1-b) ; that is $U_{n+1}\in K$.

Therefore , for all $n\ge 1$, $U_n\in K$.

b) $\left|U_{n+1}-\alpha\right|=\left|f(U_n)-\alpha\right|$ where $U_n\in K$ then , $\left|U_{n+1}-\alpha\right|\le\dfrac{1}{4}\left|U_n-\alpha\right|$.

c) Proof by induction :

$1<\alpha<2$ and $1<U_1<2$ then , $-1<U_1-\alpha<1$ and $\left|U_1-\alpha\right|<1=\left(\dfrac{1}{4}\right)^{1-1}$.

If , for a certain all $n\ge 1$, $\left|U_n-\alpha\right|\le\left(\dfrac{1}{4}\right)^{n-1}$,

$$\left|U_{n+1}-\alpha\right|=\left|f(U_n)-\alpha\right|\le\frac{1}{4}\left|U_n-\alpha\right|\le\frac{1}{4}\times\left(\frac{1}{4}\right)^{n-1}=\left(\frac{1}{4}\right)^{n}$$

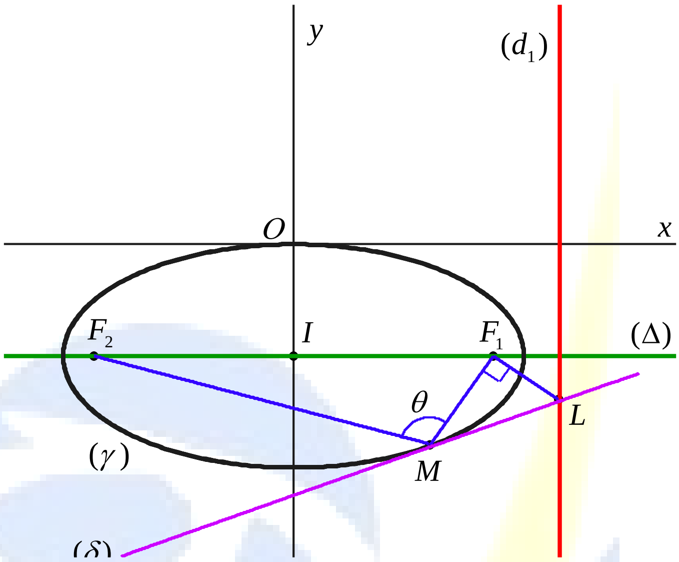
Therefore , for all $n\ge 1$, $\left|U_n-\alpha\right|\le\left(\dfrac{1}{4}\right)^{n-1}$.

$\lim\limits_{n\to+\infty}\left(\dfrac{1}{4}\right)^{n-1}=0$ then , $\lim\limits_{n\to+\infty}\left|U_n-\alpha\right|=0$; $\lim\limits_{n\to+\infty}U_n=\alpha$. Consequently , (U_n) converges to α .

Exercise 4 (9 points)

Consider the ellipse (γ) of equation $\dfrac{x^2}{4}+(y+1)^2=1$.

1- For the ellipse (γ) , the center is $I(0\,;-1)$ and the focal axis is the straight line (Δ) of equation $y=-1$.
$a=2$, $b=1$ then , the vertices of (γ) are : $(2\,;-1)$, $(-2\,;-1)$, $(0\,;0)$, $(0\,;-2)$.
Drawing (γ) .



2- The area of the domain interior to (γ) is $S=\pi ab=2\pi$ *units of area* .

The equation $\dfrac{x^2}{4}+(y+1)^2=1$ can be written as $y=-1\pm\dfrac{\sqrt{4-x^2}}{2}$ where $x\in[-2\,;2]$ then ,

$$\frac{S}{4}=\int_0^2\left(\frac{\sqrt{4-x^2}}{2}\right)dx \text{ units of area .}$$

Therefore $\displaystyle\int_0^2\sqrt{4-x^2}\,dx=\frac{S}{2}=\pi$.

3- a) An equation of (δ) is $\dfrac{\alpha x}{4}+(\beta+1)(y+1)=1$.

(δ) cuts the directrix (d_1) of equation $x=\dfrac{4}{\sqrt3}$ at $L(\dfrac{4}{\sqrt3}\,;\dfrac{\sqrt3-\alpha}{\sqrt3(\beta+1)}-1)$.

$\overrightarrow{F_1M}(\alpha-\sqrt3\,;\beta+1)$ and $\overrightarrow{F_1L}(\dfrac{1}{\sqrt3}\,;\dfrac{\sqrt3-\alpha}{\sqrt3(\beta+1)})$ then $\overrightarrow{F_1M}\cdot\overrightarrow{F_1L}=0$ and the angle $L\hat{F}M$ is right .

b) M being given on (γ), the perpendicular to (F_1M) at F_1 cuts (d_1) at a point L such that (ML) is the tangent to (γ) at M .

4- a) $d(M\,;(d_1))=\left|\alpha-\dfrac{4}{\sqrt3}\right|=\dfrac{4}{\sqrt3}-\alpha$ then , $MF_1=e\,d(M\,;(d_1))=\dfrac{\sqrt3}{2}(\dfrac{4}{\sqrt3}-\alpha)=2-\dfrac{\sqrt3}{2}\alpha$ and

$MF_2=2a-MF_1=2+\dfrac{\sqrt3}{2}\alpha$.

b) $\overrightarrow{MF_1}(\sqrt3-\alpha\,;-1-\beta)$ and $\overrightarrow{MF_2}(-\sqrt3-\alpha\,;-1-\beta)$ then ,

$$\cos\theta = \cos(\overrightarrow{MF_1}\ ;\ \overrightarrow{MF_2}) = \frac{\overrightarrow{MF_1}\cdot\overrightarrow{MF_2}}{MF_1 \times MF_2} = \frac{\alpha^2 - 3 + (\beta+1)^2}{\left(2-\frac{\sqrt{3}}{2}\alpha\right)\left(2-\frac{\sqrt{3}}{2}\alpha\right)} = \frac{\alpha^2 - 3 + 1 - \frac{\alpha^2}{4}}{4 - 3\frac{\alpha^2}{4}} = \frac{3\alpha^2 - 8}{16 - 3\alpha^2}.$$

If M is one of the vertices on the non focal axis of (γ) then , $\alpha = 0$ and $\cos\theta = -\frac{1}{2}$; therefore

$\theta = \frac{2\pi}{3}$ *radians* .

c) The points of (γ) of ordinate -1 do not belong to the circle of diameter $[F_1F_2]$.

The points of (γ) that belong to the circle of diameter $[F_1F_2]$ are the points $M(\alpha\ ;\ \beta)$ where $\beta \neq -1$

such that $F_1\hat{M}F_2$ is right ; they are the points $M(\alpha\ ;\ \beta)$ such that $\cos\theta = \frac{3\alpha^2 - 8}{16 - 3\alpha^2} = 0$; $3\alpha^2 - 8 = 0$;

$\alpha = -2\sqrt{\frac{2}{3}}$ or $\alpha = 2\sqrt{\frac{2}{3}}$.

Therefore , the points of (γ) that are on the circle of diameter $[F_1F_2]$ are the 2 points of abscissas $-2\sqrt{\frac{2}{3}}$ and the 2 points of abscissas $2\sqrt{\frac{2}{3}}$.

<u>Exercise 5</u> **(9 points)**

1- The similitude S of center A transforms K into H .

The triangle AKH is semi equilateral then , the ratio of S is $\frac{AH}{AK} = 2$ and an angle of S is $\frac{\pi}{3}$.

The similitude S' transforms B into H and H into K where

$\frac{HK}{BH} = \sin\frac{\pi}{6} = \frac{1}{2}$ and $(\overrightarrow{BH}\ ;\ \overrightarrow{HK}) = (\overrightarrow{BH}\ ;\ \overrightarrow{HA}) + (\overrightarrow{HA}\ ;\ \overrightarrow{HK}) = \frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3}\ (2\pi)$ then , the ratio of S' is $\frac{1}{2}$ and an angle of S' is $\frac{2\pi}{3}$.

2- a) $T(H) = S \circ S'(H) = S(S'(H)) = S(K) = H$.

$T = S \circ S'$ where S and S' are two similitudes of ratios 2 and $\frac{1}{2}$ of product 1 and angles $\frac{\pi}{3}$ and $\frac{2\pi}{3}$ of sum π then T is a similitude of ratio 1 , angle π that keeps H invariant .

Therefore , T is the central symmetry of center H .

b) $T(C) = A$ then $S(S'(C)) = A$; that is $S(S'(C)) = S(A)$ and $S'(C) = A$.

3- a) $AK = \frac{1}{2}AH = \frac{1}{4}AC$ then , $\overrightarrow{AK} = \frac{1}{4}\overrightarrow{AB} = \frac{1}{2}\overrightarrow{AI}$ then , K is the mid point of $[AI]$.

$S'(C) = A$, $S'(H) = K$ and A is the symmetric of C with respect to H then , $S'(A)$ is the symmetric of $S'(C) = A$ with respect to $S'(H) = K$; therefore $S'(A) = I$.

b) $S'(A) = I$, $S'(B) = H$ and I is the mid point of $[AB]$ then , $S'(I) = J$, the mid point of $[IH]$.

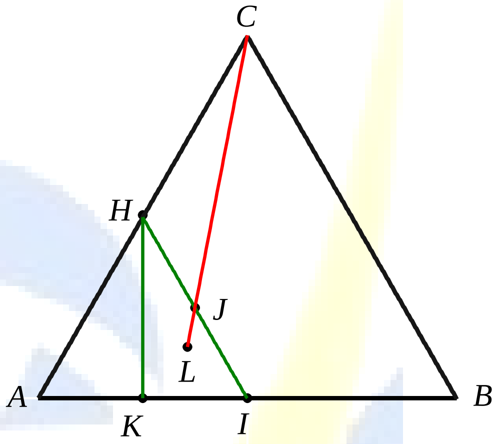
4- a) S' is a similitude of ratio $\frac{1}{2}$ and angle $\frac{2\pi}{3}$ and $f = S' \circ S' \circ S'$ then , f is a similitude of ratio $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$ and angle $\frac{2\pi}{3} \times 3 = 2\pi$; therefore f is a dilation of ratio $\frac{1}{8}$ having same center as S'.

b) $f(C) = S' \circ S' \circ S'(C) = S' \circ S'(A) = S'(I) = J$.

f is a dilation of ratio $\frac{1}{8}$ such that $f(C) = J$ then ,

its center is the point L such that $\overrightarrow{LJ} = \frac{1}{8}\overrightarrow{LC}$.

S' and f have the same center then , L is the center of S'.



5- The plane is referred to the direct orthonormal system $(A\,;\vec{u}\,;\vec{v})$ where $\vec{u} = \frac{1}{4}\overrightarrow{AB}$.

a) The complex relation of the similitude S' of ratio $\frac{1}{2}$ and angle $\frac{2\pi}{3}$ is of the form $z' = \frac{1}{2}e^{i\frac{2\pi}{3}}z + b$.

$A(0\,;0)$, $I(2\,;0)$ and $S'(A) = I$ then $2 = b$; therefore $z' = \frac{-1+\sqrt{3}\,i}{4}z + 2$.

b) $J = S'(I)$ then , the affix of J is $z_J = \frac{-1+\sqrt{3}\,i}{4} \times 2 + 2 = \frac{3+\sqrt{3}\,i}{2}$.

The affix of the center L of S' is such that $z_L = \frac{-1+\sqrt{3}\,i}{4}z_L + 2$ then ,

$z_L = \frac{8}{5-\sqrt{3}\,i} = \frac{10+2\sqrt{3}\,i}{7}$.

Exercise 6 **(11 points)**

A- 1- a) $\overrightarrow{MN} = (x'-x)\vec{i} + (y'-y)\vec{j} = -2x\,\vec{i} - 2x\,\vec{j} = -2x(\vec{i} + \vec{j})$ then , $\overrightarrow{MN}$ is collinear to $\vec{i} + \vec{j}$.

The abscissa of the mid point P of $[MN]$ is $\dfrac{x'+x}{2} = 0$ then , P belongs to the axis of ordinates .

b) Let (d) be the straight line of equation $y = x$ having $\vec{u} = \vec{i} + \vec{j}$ as a direction vector .
M being any point of plane , the parallel to (d) drawn through M cuts the axis of ordinates at point P ; the symmetric of M with respect to P is the image N of M by T .

2- a) Let $M(0\,;\,y)$ be any point of the axis of ordinates ; the coordinates of the image of M by T are $x'=0$ and $y'=y$ then , $M'=M$ and M is invariant by T .

b) Let (d) be a straight line of director coefficient a ; an equation of (d) is of the form $y = ax+b$ where $b \in IR$.
An equation of the image of (d) by T is $-2x + y = a\,x + b$; $y = (a+2)\,x + b$; therefore , the image of (d) by T is a straight line (d') of equation $y = (a+2)\,x + b$.
The straight lines (d) and (d') intersect at the point $(0\,;\,b)$ which is on the axis of ordinates .

B- 1- a) g is defined on $]-\infty\,;\,0]$ by $g(x) = x - 1 + 2e^{x}$.

$\lim\limits_{x\to-\infty} e^{x} = 0$ then , $\lim\limits_{x\to-\infty} g(x) = -\infty$.

$g'(x) = 1 + 2e^{x}$.

Table of variations of g .

x	$-\infty$		0
$g'(x)$		$+$	2
$g(x)$	$-\infty$	$\nearrow$	1

b) h is defined on $[0\,;\,+\infty[$ by $h(x) = x - 1 + 2e^{-x}$.

$\lim\limits_{x\to+\infty} e^{-x} = 0$ then , $\lim\limits_{x\to+\infty} h(x) = +\infty$.

$h'(x) = 1 - 2e^{-x}$.

Table of variations of h .

x	0		$\ell n2$		$+\infty$
$h'(x)$	-1	$-$	0	$+$	
$h(x)$	1	$\searrow$	$\ell n2$	$\nearrow$	$+\infty$

2- a) The relations $x' = -x$ and $y' = -2x + y$ are equivalent to $x = -x'$ and $y = y' - 2x'$.
$M(x\,;\,y)$ belongs to (C_1) if and only if $y = x - 1 + 2e^{x}$; that is $y' - 2x' = -x' - 1 + 2e^{-x'}$;
$y' = x' - 1 + 2e^{-x'}$; therefore an equation of the image of (C_1) by T is $y = x - 1 + 2e^{-x}$.
Therefore , (C_2) is the image of (C_1) by T .

b) $\lim\limits_{x\to-\infty} \big(g(x) - (x-1)\big) = \lim\limits_{x\to-\infty} e^{x} = 0$ then , the straight line (δ) is asymptote to (C_1) at $-\infty$;

$\lim\limits_{x\to+\infty} \big(h(x) - (x-1)\big) = \lim\limits_{x\to+\infty} e^{-x} = 0$ then , the straight line (δ) is asymptote to (C_2) at $+\infty$.

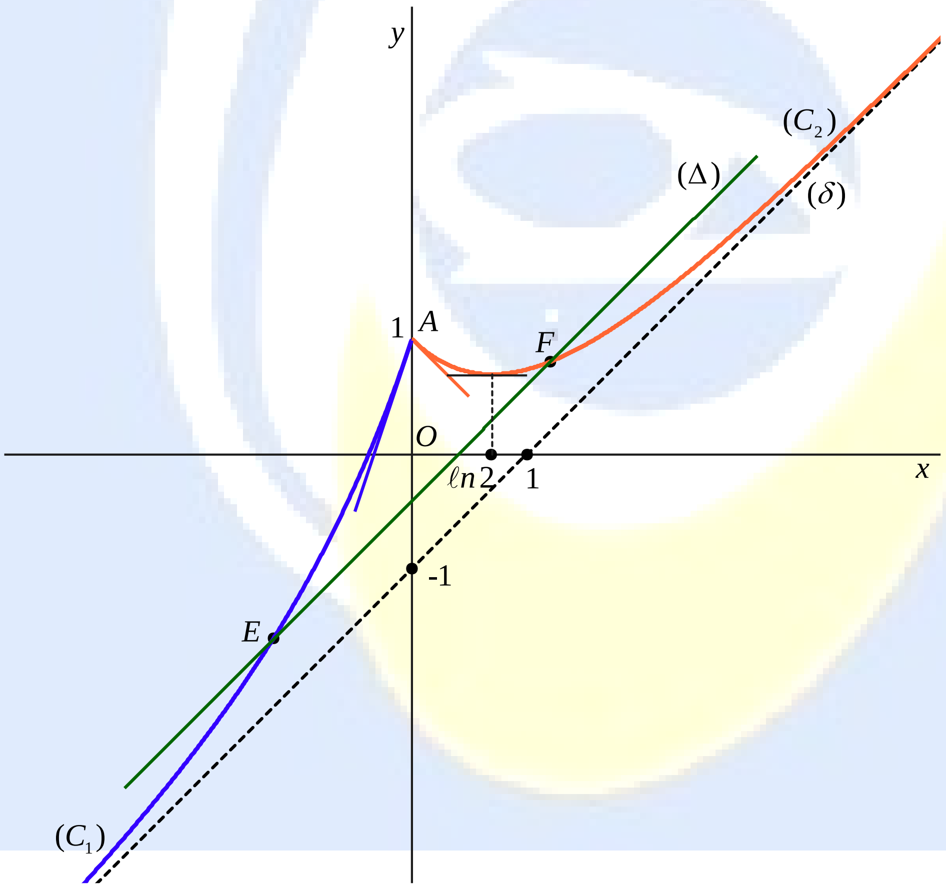
c) For all $x \in]-\infty\,;\,0]$, $g(x)-(x-1)=2e^{x}>0$ and , for all $x \in [0\,;\,+\infty[$, $h(x)-(x-1)=2e^{-x}>0$ then , each of (C_1) and (C_2) lies above (δ).

3- The function f is defined on IR by $f(x)=x-1+2e^{-|x|}$.

a) $f(x)=\begin{cases} x-1+2e^{x}=g(x) & if\ \ x\in]-\infty\,;\,0] \\ x-1+2e^{-x}=h(x) & if\ \ x\in[0\,;\,+\infty[\end{cases}$; therefore (C) is the union of (C_1) and (C_2).

b) The semi tangent to (C) at the point A of abscissa 0 from the left has the slope $g_{\ell}'(0)=2$ while
the semi tangent to (C) at the point A of abscissa 0 from the right has the slope $h_{r}'(0)=-1$.

c) Drawing $(C)=(C_1)\cup(C_2)$ (***Graph unit 2 cm***) .



4- (Δ) is the straight line of equation $y = x-1+2m$ where $m \in]0\,;1[$.

a) The equation $f(x) = x-1+2m$ is equivalent to $e^{-|x|} = m$; that is $-|x| = \ell n(m)$;

$|x| = -\ell n(m)$;

$x = \ell n(m)$ or $x = -\ell n(m)$.

Therefore , (Δ) cuts (C) at two points E and F of abscissas $\ell n(m)$ and $-\ell n(m)$.

For all $m \in]0\,;1[$, $\ell n(m) < 0$ then , $E \in (C_1)$ and $-\ell n(m) > 0$ then , $F \in (C_2)$.

b) The coordinates of E are $x = \ell n(m)$ and $y = g(\ell n(m)) = \ell n(m) - 1 + 2m$.

The coordinates of F are $x = -\ell n(m)$ and $y = h(-\ell n(m)) = -\ell n(m) - 1 + 2m$.

The coordinates of the image of E by T are $x' = -x = -\ell n(m)$ and

$y' = -2x + y = -2\ell n(m) + \ell n(m) - 1 + 2m = -\ell n(m) - 1 + 2m$.

Therefore $T(E) = F$.

5- It is given that the tangent (t_1) at E to (C_1) and the tangent (t_2) at F to (C_2) intersect at a point L belonging to the axis of ordinates .

(t_1) is the straight line (LE) where $T(E) = F$ and $T(L) = L$ since L is on the axis of ordinates then , the image (t_1) , which is a straight line , is the straight line (LF) which is the straight line (t_2) .