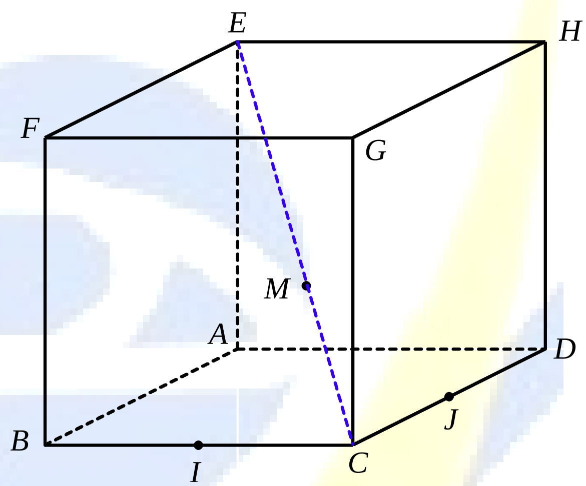
Entrance Exam 2018 - 2019 ***<u>Mathematics</u>*** ***Duration : 3 hours***
The distribution of grades is over 50 ***July 07 , 2018***

<u>Exercise 1</u> **(10 points)**

$ABCDEFGH$ is a cube of side 1 ; I and J are the respective mid points of $[BC]$ and $[CD]$.
Refer the space to the direct orthonormal system $(A\,;\,\overrightarrow{AB}\,,\,\overrightarrow{AD}\,,\,\overrightarrow{AE}\,)$.

1- a) Determine the coordinates of E , I and J and prove that $EI = EJ$.
   b) Deduce that , for any point M on (CE) ,
      the triangle MIJ is isosceles at M .



2- The goal of this part is to determine the position of M on (CE) for which the angle $I\hat{M}J$ is maximum .
   Let θ be the measure in radians of the angle $I\hat{M}J$.
   a) Prove that $\sin\dfrac{\theta}{2} = \dfrac{IJ}{2\,MI}$.
   b) Justify that $I\hat{M}J$ is maximum when MI is minimum.
   c) Prove that there exists a unique position M_0 of M on (CE) for which the angle $I\hat{M}J$ is maximum .

3- a) Determine the coordinates of M_0 .
   b) Verify that M_0 belongs to the segment $[CE]$.
   c) Determine , the maximum value of θ .

<u>Exercise 2</u> **(8 points)**

An urn contains 4 red balls and 2 green balls indistinguishable to the touch .
A child draws simultaneously and at random two balls from the urn .
For $n \in \{0\,;1\,;2\}$, let A_n be the event " the child got n green balls " .

1- Calculate the probabilities $p(A_0)$, $p(A_1)$ and $p(A_2)$.

2- Knowing that the child has at least one red ball , calculate the probability that he has two red balls .

3- After the first draw, there remains 4 balls in the urn from which the child draws two balls simultaneously.
a) Knowing that the first two balls drawn were red , calculate the probability that the last two balls are also red .
b) Calculate the probability that the child got the 4 red balls in the two draws .
c) Calculate the probability that, in the second draw, the child gets 2 red balls .

4- Consider the event E : " it took exactly the two draws to draw the two green balls from the urn ".
Prove that $p(E)=\frac{1}{3}$.

Exercise 3 **(10 points)**

The plane (P) is referred to a direct orthonormal system $(O\ ;\vec{u}\ ,\vec{v})$.
Let f be the mapping of $(P)-\{O\}$ into (P) that , to each point M of affix z ($z\neq 0$) , associates the point N of affix z' defined by : $z'=z-\frac{1}{z}$.

1- a) Determine the points whose image by f is O.
b) Determine the point whose image by f is the point E of affix $2i$.

2- Prove that any point N of plane (P), except two points to be determined , has two antecedents (pre-images) by f .

3- Let $z=r\,e^{i\theta}$ ($r>0$) be the exponential form of the affix z of a point M .
a) Calculate the coordinates x' and y' of the image N of M in terms of r and θ .
b) Prove that , as M varies on the circle (C) of centre O and radius 2 , N varies on an ellipse (E) to be determined with its eccentricity .

c) Prove that , as M varies on the semi straight line $]Ot)$ of direction vector $\vec{u}+\vec{v}$, N varies on a hyperbola (H) of center O to be determined with its eccentricity.
4- a) Prove that (E) and (H) have the same foci F and F' to be determined .
b) Draw (E) and (H) in the same system . (***Graph unit :*** 2 ***cm***)

<u>Exercise 4</u> **(14 points)**

Let f be a function defined and <u>two times differentiable</u> on the set IR of real numbers , such that

$$\begin{cases} f'(0)=1 \\ For\ all\ x\ in\ IR\ ,\ \ (f'(x))^2-(f(x))^2=1 \quad (1) \end{cases}$$

Let (C) be the representative curve of f in an orthonormal system $(O\ ;\ \vec{i}\ ,\ \vec{j})$. (***Graph unit : 1 cm***)

1- Calculate $f(0)$ and prove that (C) is tangent to the straight line (d) of equation $y = x$.

2- a) Prove that , for all x in IR , $f'(x) \neq 0$.
b) Deduce that , for all real numbers a and b , $f'(a)\times f'(b) > 0$.
c) Calculate $f'(x)\times f'(0)$. Deduce that , for all x in IR , $f'(x) > 0$.
d) By differentiating the two members of the relation (1), prove that for all x in IR , $f''(x) = f(x)$.

3- Let g and h be the functions defined on IR by $g = f' + f$ and $h = f' - f$.
a) Calculate $g(0)$ and $h(0)$.
b) Justify that g and h are differentiable on IR and prove that $g' = g$ and $h' = -h$.
c) Deduce the functions g and h , then prove that , for all x in IR , $f(x) = \dfrac{e^x - e^{-x}}{2}$.

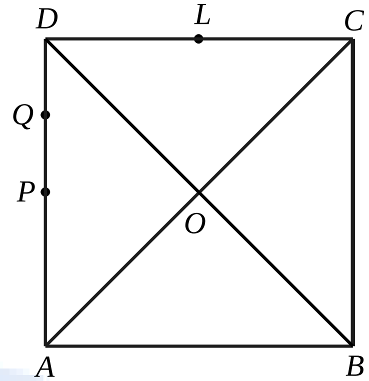
4- a) Set up the table of variations of f .
b) Prove that , for all values of λ in IR , the equation $f(x) = \lambda$ has a unique solution , then calculate this solution in terms of λ .
c) Draw (C).

5- a) Prove that f has an inverse function f^{-1} whose domain of definition is to be determined .
b) Draw the representative curve (C') of f^{-1} in the same system as (C).

6- Denote by α the ordinate of the point A of (C) with abscissa 2 .
Let (Δ) be the straight line of slope -1 passing through A .
a) Determine the coordinates of the point A' where (Δ) cuts (C') in terms of α .
b) Prove that the area of the triangle OAA' is $S = \dfrac{\alpha^2 - 4}{2}\ cm^2$.
c) Deduce the area of the domain bounded by (C), (C'), (Δ) and lying above the axis of abscissas .

Exercise 5 **(8 points)**

$ABCD$ is a direct square of center O such that $AB = 4$.
Let L , P and Q be the mid points of $[DC]$, $[AD]$ and $[DP]$ respectively.
Let S be the similitude that transforms A into O and B into L.



1- Determine the ratio and the angle of S.

2- a) Determine the image of each of the straight lines (BC) and (AC) by S.
b) Deduce $S(C)$.

3- Consider the similitudes $S_1(D\,;\frac{\sqrt{2}}{2}\,;\frac{\pi}{4})$ and $S_2(O\,;\frac{\sqrt{2}}{2}\,;\frac{\pi}{4})$.
a) Determine $S_2 \circ S_1(A)$ and prove that $S_2 \circ S_1 = S$.
b) Deduce $S(D)$ and prove that $S(L) = Q$.

4- Denote by I the center of S and by h the transformation $S \circ S$.
a) Determine $h(B)$ and $h(C)$.
b) Justify that h is a dilation to be determined .
c) Deduce that I is the point of intersection of the two straight lines (BQ) and (CP).
d) Prove that I belongs to the circle (γ) of diameter $[DC]$ and that (BQ) is the tangent to (γ) at I .

Concours d'entrée 2018 - 2019 **Mathematics Solution** **Durée : 3 heures**
La distribution des notes est sur 50 **7 Juillet 2018**

Exercise 1 (10 points)

1- a) In the system $(A\,;\overrightarrow{AB}\,,\overrightarrow{AD}\,;\overrightarrow{AE})$, $E(0\,;0\,;1)$.
$\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{AD}$, then $C(1\,;1\,;0)$.

I is the mid point of $[BC]$ where $B(1\,;0\,;0)$ and $C(1\,;1\,;0)$, then $I(1\,;\frac{1}{2}\,;0)$.

J is the mid point of $[CD]$ where $C(1\,;1\,;0)$ and $D(0\,;1\,;0)$, then $J(\frac{1}{2}\,;1\,;0)$.

Therefore , $EI=EJ=\frac{\sqrt{5}}{2}$.

b) $CI=CJ=\frac{1}{2}$ and $EI=EJ$, then C and E belong to the mediator plane (L) of $[IJ]$, then (CE) lies in the plane (L) ; therefore M belongs to (L) and $MI=MJ$.
Consequently , the triangle MIJ is isosceles at M .

2- a) The triangle MIJ is isosceles at M and θ is the measure of the angle $I\hat{M}J$, then $\frac{\theta}{2}$ is the measure of the angle $I\hat{M}K$ where K is the mid point of $[IJ]$.

The triangle MIK is right at K , then

$$\sin\frac{\theta}{2}=\frac{IK}{MI}=\frac{IJ}{2\,MI} .$$

b) $I\hat{M}J$ is maximum when $\frac{\theta}{2}$ is maximum ;

that is when $\sin\frac{\theta}{2}$ is maximum since $0<\frac{\theta}{2}<\frac{\pi}{2}$.

Therefore , $I\hat{M}J$ is maximum when MI is minimum since IJ is constant .

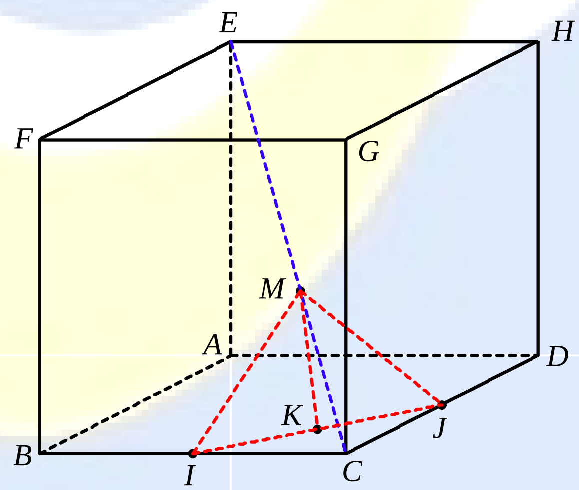


Figure 10

c) As M varies on (CE), I remaining fixed ,
MI is minimum when M is at M_0 , the orthogonal projection of I on (EC) .

3- a) $C(1\,;1\,;0)$, $\overrightarrow{CE}(-1\,;-1\,;1)$ and $\overrightarrow{CM}=\lambda\,\overrightarrow{CE}$, then a system of parametric equations of (CE) is : $(x=-\lambda+1\ ;\ y=-\lambda+1\ ;\ z=\lambda)$ where $\lambda\in IR$.

$M\in(CE)$, then $M(-\lambda+1\,;-\lambda+1\,;\lambda)$ and $\overrightarrow{IM}(-\lambda\ ;-\lambda+\frac{1}{2}\ ;\lambda)$

M_0 is such that $\overrightarrow{CE}\,.\,\overrightarrow{IM_0}=0$; that is $\lambda+\lambda-\frac{1}{2}+\lambda=0$; therefore $\lambda=\frac{1}{6}$ and $M_0(\frac{5}{6};\frac{5}{6};\frac{1}{6})$.

b) $\lambda=\frac{1}{6}$, then $\overrightarrow{CM_0}=\frac{1}{6}\,\overrightarrow{CE}$; therefore , M_0 belongs to the segment $[CE]$.

c) The maximum of θ is such that $\sin\frac{\theta}{2}=\frac{IJ}{2\,IM_0}$ where $IJ=\frac{\sqrt{2}}{2}$ and $IM_0=\sqrt{\frac{1}{36}+\frac{1}{9}+\frac{1}{36}}=\frac{\sqrt{6}}{6}$;

therefore $\sin\frac{\theta}{2}=\frac{\sqrt{3}}{2}$ with $0<\frac{\theta}{2}<\frac{\pi}{2}$, then $\frac{\theta}{2}=\frac{\pi}{3}$ rad ; $\theta=\frac{2\pi}{3}$ rad .

<u>Exercise 2</u> **(8 points)**

1- The sample space is equibrobable and consists of ${}_6C_2$ possible outcomes .

$p(A_0)=\frac{{}_4C_2}{{}_6C_2}=\frac{6}{15}=\frac{2}{5}$; $p(A_1)=\frac{{}_4C_1\times{}_2C_1}{{}_6C_2}=\frac{8}{15}$ and $p(A_2)=\frac{{}_2C_2}{{}_6C_2}=\frac{1}{15}$.

2- Let L be the event: " the child has at least one red ball " is the opposite of the event " no red ball is drawn "

which is the event A_2 , then $p(L)=1-p(A_2)=\frac{14}{15}$.

The required probability is $p\left(A_0/L\right)=\frac{p(A_0\cap L)}{p(L)}=\frac{p(A_0)}{p(L)}=\frac{6}{14}=\frac{3}{7}$.

3- After the first draw, there remains 4 balls in the urn from which the child draws two new balls .

a) If the first two balls were red , then , for the second draw , the urn will contain 2 red balls and 2 green balls ; therefore , the required probability is $p_1=\frac{{}_2C_2}{{}_4C_2}=\frac{1}{6}$.

b) The required probability is $p_2 = p(A_0)\times p_1 = \dfrac{2}{5}\times\dfrac{1}{6} = \dfrac{1}{15}$.

c) Let B be the event : " the child get 2 red balls in the second draw "

$$p(B) = p(B\cap A_0) + p(B\cap A_1) + p(B\cap A_2)$$
$$= p(A_0)\times p(B/A_0) + p(A_1)\times p(B/A_1) + p(A_2)\times p(B/A_2).$$
$$= \frac{1}{15} + \frac{8}{15}\times\frac{{}_3C_1\times{}_1C_1}{{}_4C_2} + \frac{1}{15}\times 1 = \frac{1}{15} + \frac{4}{15} + \frac{1}{15} = \frac{6}{15} = \frac{2}{5}\ .$$

4- The event E is realized when either one of the following incompatible events is :
" he draws one green ball in each draw " ;
" he draws no green ball in the first draw and two green balls in the second " .

Therefore $p(E) = p(A_1)\times\dfrac{1\times{}_3C_1}{{}_4C_2} + p(A_0)\times\dfrac{{}_2C_2}{{}_4C_2} = \dfrac{8}{15}\times\dfrac{1}{2} + \dfrac{2}{5}\times\dfrac{1}{6} = \dfrac{4}{15} + \dfrac{1}{15} = \dfrac{1}{3}$.

Exercise 3 **(10 points)**

1- a) The equation $z - \dfrac{1}{z} = 0$ is equivalent to $z^2 = 1$; that is $z = -1$ or $z = 1$ then , the points whose image by f is the origin O are the points with affixes $z = -1$ and $z = 1$.

b) The equation $z - \dfrac{1}{z} = 2i$ is equivalent to $z^2 - 2iz - 1 = 0$; that is $(z-i)^2 = 0$; $z = i$ then , the point whose image by f is the point E is the point with affix $z = i$.

2- The affixes of the antecedents of a point N of affix z' are the solutions of the equation $z - \dfrac{1}{z} = z'$ which is equivalent to $z^2 - z'z - 1 = 0$.

The equation $z^2 - z'z - 1 = 0$, which is of the second degree , has two roots except when $\Delta = 0$; that is $z'^2 + 4 = 0$; $z = 2i$ or $z = -2i$.

Therefore , any point N of plane (P), except $E(2i)$ and $E'(-2i)$, has two antecedents by f .

3- Let $z = re^{i\theta}$ ($r > 0$) be the exponential form of z .

a) The affix of N is $z' = z - \dfrac{1}{z} = re^{i\theta} - \dfrac{1}{re^{i\theta}} = re^{i\theta} - \dfrac{1}{r}e^{-i\theta} = r(\cos\theta + i\sin\theta) - \dfrac{1}{r}(\cos\theta - i\sin\theta)$;

$z' = \left(r - \dfrac{1}{r}\right)\cos\theta + i\left(r + \dfrac{1}{r}\right)\sin\theta$; therefore $x' = \left(r - \dfrac{1}{r}\right)\cos\theta$ and $y' = \left(r + \dfrac{1}{r}\right)\sin\theta$.

b) M varies on the circle (C) of centre O and radius 2 , then $OM = r = 2$. therefore the coordinates of N become : $x' = \dfrac{3}{2}\cos\theta$ and $y' = \dfrac{5}{2}\sin\theta$.

Therefore , N varies on the ellipse (E) of equation $\dfrac{x^2}{\frac{9}{4}}+\dfrac{y^2}{\frac{25}{4}}=1$.

For the ellipse (E), $a=\dfrac{5}{2}$ and $b=\dfrac{3}{2}$ then $c=\sqrt{a^2-b^2}=2$ and the eccentricity is $e=\dfrac{c}{a}=\dfrac{4}{5}$.

c) M varies on the semi straight line $]Ot)$ of direction vector $\overrightarrow{u}+\overrightarrow{v}$, then $\theta=\dfrac{\pi}{4}$; therefore the coordinates of N become : $x'=\dfrac{\sqrt{2}}{2}\left(r-\dfrac{1}{r}\right)$ and $y'=\dfrac{\sqrt{2}}{2}\left(r+\dfrac{1}{r}\right)$.

$x'^2=\dfrac{1}{2}\left(r^2+\dfrac{1}{r^2}-2\right)$ and $y'^2=\dfrac{1}{2}\left(r^2+\dfrac{1}{r^2}+2\right)$, then $y'^2-x'^2=2$.

Therefore , N varies on the equilateral hyperbola (H) of equation $y^2-x^2=2$.

(H) is an equilateral hyperbola , then its eccentricity is $e'=\sqrt{2}$.

4- a) The center of (E) is O ; the focal axis is the axis of ordinates ; $c=2$, then the foci of (E) are the points $F(0\,;\,2)$ and $F'(0\,;\,-2)$.

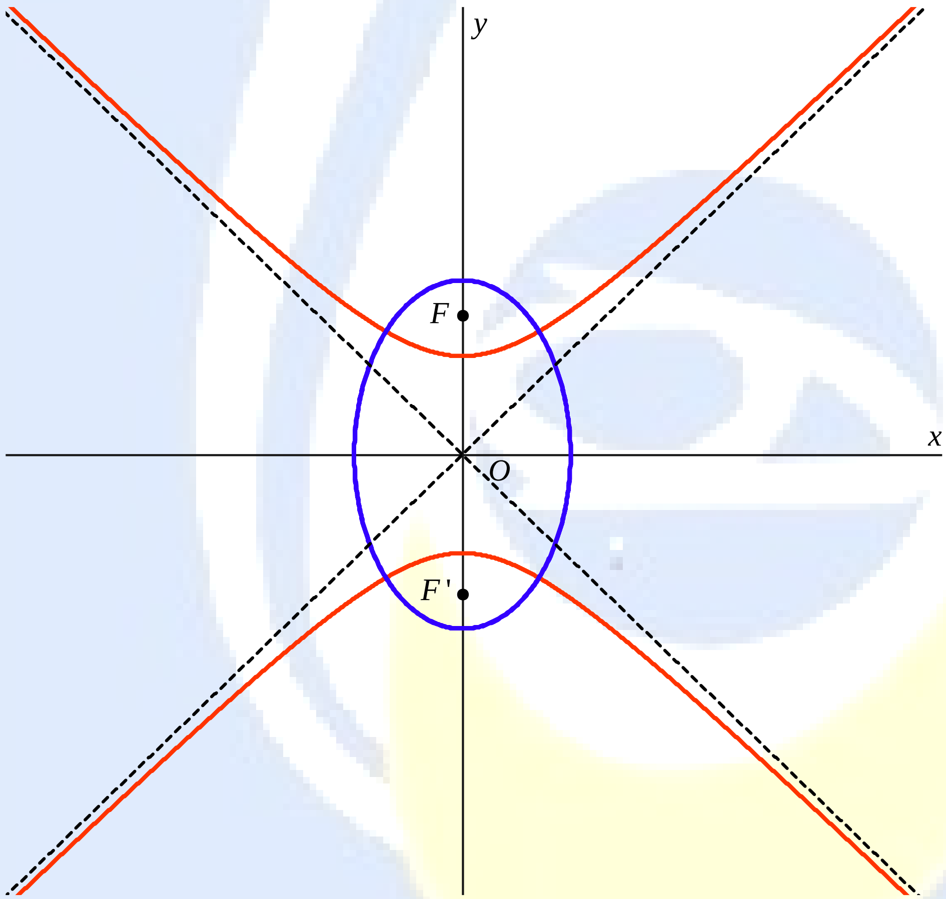
The center of (H) is O ; the focal axis is the axis of ordinates , $a=b=\sqrt{2}$ then , $c=a\sqrt{2}=2$ and the foci of (H) are also the points F and F'.

b) The vertices of (E) are $A(0\,;\frac{5}{2})$, $A'(0\,;-\frac{5}{2})$, $B(\frac{3}{2}\,;0)$ and $B'(-\frac{3}{2}\,;0)$.

The vertices of (H) are $C(0\,;\sqrt{2})$ and $C'(0\,;-\sqrt{2})$.

The asymptotes of (H) are the straight lines of equations $y=x$ and $y=-x$.

Drawing (E) and (H) in the same system .



Exercise 4 **(14 points)**

1- By applying the relation (1) to the real number 0, we find $f(0)=0$.
$f(0)=0$ and $f'(0)=1$, then an equation of the tangent to (C) at the point $(0\,;1)$ is $y=x$.

2- a) The relation (1) gives , $(f'(x))^2=1+(f(x))^2\neq 0$ then , for all x in IR, $f'(x)\neq 0$.

b) The function f' is differentiable , then it is continuous on IR .
If there exists two real numbers a and b such that $f'(a)f'(b)<0$, then there exists a real number
x_0 belonging to $]a\,;b[$ $(a<b)$ such that $f'(x_0)=0$ which is impossible since for all x in IR , $f'(x)\neq 0$. Therefore , for all real numbers a and b, $f'(a)f'(b)>0$.

c) $f'(0)=1$, then $f'(x)f'(0)=f'(x)$, then for all x in IR, $f'(x)>0$.

d) By differentiating the two members of the relation (1), we find : $f'(x)\times f''(x)-f(x)\times f'(x)=0$
where $f'(x)\neq 0$, then for all x in IR, $f''(x)-f(x)=0$; that is $f''(x)=f(x)$.

3- The functions g and h are defined on IR , by $g=f'+f$ and $h=f'-f$.

a) $g(0)=f'(0)+f(0)=1$ and $h(0)=f'(0)-f(0)=1$.

b) The two functions f and f' are differentiable on IR, then g and h are differentiable on IR .
$g'=(f+f')'=f'+f''=f'+f=g$ and $h'=(f-f')'=f'-f''=f'-f=-h$.

c) $g'=g$, then g is a solution of the differential equation $y'-y=0$, then $g(x)=Ce^x$.
$g(0)=1$, then $C=1$; therefore , $g(x)=e^x$.
Similarly , $h(x)=e^{-x}$.

$g=f'+f$ and $h=f'-f$ give $g-h=2f$, then for all x in IR, $f(x)=\dfrac{e^x-e^{-x}}{2}$.

4- a) $\lim\limits_{x\to+\infty} e^x=+\infty$ and $\lim\limits_{x\to-\infty} e^x=0$;
then $\lim\limits_{x\to+\infty} f(x)=+\infty$.

Similarly , $\lim\limits_{x\to-\infty} f(x)=-\infty$.

$f'(x)>0$.

Table of variations of f .

x	$-\infty$		0		$+\infty$
$f'(x)$		$+$	0	$+$	
$f(x)$	$-\infty$	$\nearrow$	0	$\nearrow$	$+\infty$

Figure 25

b) f is continuous and strictly increasing and $f(IR) = IR$, then for all values of λ in IR , the equation

$f(x) = \lambda$ has a unique solution.

The equation $f(x) = \lambda$ is equivalent to $e^{x} - e^{-x} = 2\lambda$; that is $e^{2x} - 2\lambda e^{x} - 1 = 0$ with $e^{x} > 0$

The quadratic equation $t^2 - 2\lambda t - 1 = 0$ of discriminant $\Delta' = \lambda^2 + 1 > 0$ has only one positive root

$t = \lambda + \sqrt{\lambda^2 + 1}$, then $e^{x} = \lambda + \sqrt{\lambda^2 + 1}$; therefore $x = \lambda n(\lambda + \sqrt{\lambda^2 + 1})$

c) $\lambda im_{x \to \pm\infty} \frac{f(x)}{x} = \lambda im_{x \to \pm\infty} \left(\frac{e^{x}}{x} - \frac{e^{-x}}{x} \right) = +\infty$, then (C) has at $+\infty$ and at $-\infty$ an asymptotic direction parallel to the axis of ordinates .

Drawing (C) .

5- a) f is continuous and strictly increasing , then that f has an inverse function f^{-1} defined on $f(IR) = IR$.

b) Drawing (C') (by symmetry with respect to the straight line (d) of equation $y = x$) .

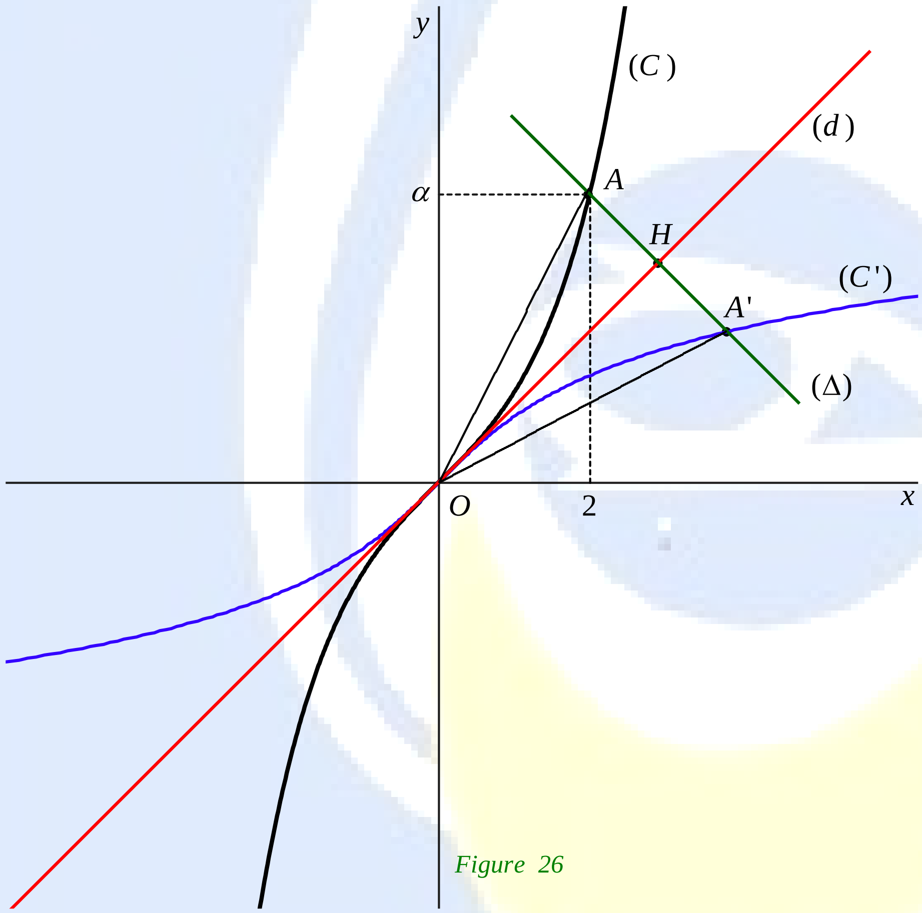


Figure 26

6- $A(2\,;\alpha)$ belongs to (C) ; (Δ) is the straight line of slope -1 passing through A .

a) (C) and (C') are symmetric with respect to (d) ; then the straight line (Δ) which is the perpendicular to (d) passing through A will cut (C') at the point A' symmetric of A with respect to (d), then $A'(\alpha\,;2)$.

b) The mid point of $[AA']$ is $H(\frac{\alpha+2}{2}\,;\,\frac{\alpha+2}{2})$.

The area of the triangle OAA' is $S = OH \times AH = \quad cm^2$ where :

$OH = \frac{\alpha+2}{\sqrt{2}}$ and $AH = \frac{\alpha-2}{\sqrt{2}}$, $(\alpha \approx 3.6 > 2)$; therefore $S = \frac{\alpha^2-4}{2}\ \ cm^2$.

c) The area of the domain bounded by (C), (C') and (Δ) is equal to $S - 2S'$ *units of area* where S' is the area of the domain bounded by (C), and the straight line (OA) lying above the axis of abscissas .

$A(2\,;\,\alpha)$, then an equation of the straight line (OA) is $y = \frac{\alpha}{2}x$.

$$S' = \int_0^2 \left(\frac{\alpha}{2}x - f(x)\right)dx = \frac{1}{2}\left[\frac{\alpha}{2}x^2 - e^x - e^{-x}\right]_0^2 = \frac{1}{2}\left(2\alpha - e^2 - e^{-2}\right) - \frac{1}{2}(-1-1)\ ;$$

$$S' = \frac{1}{2}\left(2\alpha - e^2 - e^{-2} + 2\right)\ cm^2.$$

Therefore , the required area is $A = e^2 + e^{-2} + \frac{\alpha^2 - 4\alpha - 8}{2}\ \ cm^2$.

Exercise 5 **(8 points)**

1- $S(A) = O$ and $S(B) = L$ where

$\frac{OL}{AB} = \frac{1}{2}$ and $(\overrightarrow{AB}\ ;\overrightarrow{OL}) = \frac{\pi}{2}\ \ (2\pi)$.

Therefore the ratio of S is $\frac{1}{2}$ and its angle is $\frac{\pi}{2}$.

2- S is a similitude of angle $\frac{\pi}{2}$, then any straight line and its image are perpendicular .

a) $S(B) = L$, then $S((BC)) = (DC)$.

$S(A) = O$, then $S((AC)) = (DB)$.

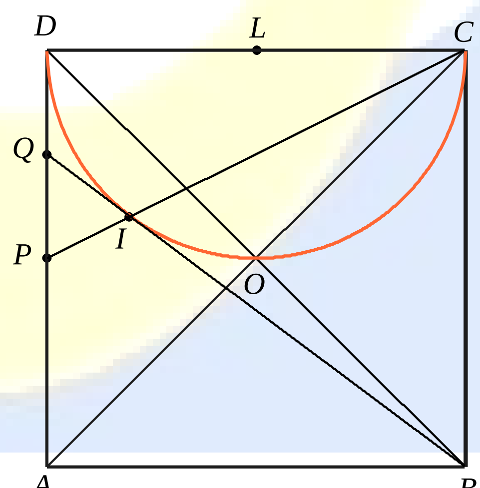


Figure 36 a

b) C is the point of intersection of (AC) and (BC) ;

$S((AC))=(DB)$ and $S((BC))=(DC)$, then the image of C is the point of intersection D of (DB) and (DC);

$S(C)=D$.

3- a) $S_1 = S(D\,;\frac{\sqrt{2}}{2}\,;\frac{\pi}{4})$ and $S_2 = S(O\,;\frac{\sqrt{2}}{2}\,;\frac{\pi}{4})$.

$S_2 \circ S_1(A)=S_2(O)=O$.

$S_2 \circ S_1$ is a similitude of ratio $\left(\frac{\sqrt{2}}{2}\right)^2=\frac{1}{2}$ and angle $2\left(\frac{\pi}{4}\right)=\frac{\pi}{2}$.

The similitudes $S_2 \circ S_1$ and S have same ratio , same angle and $S_2 \circ S_1(A)=S(A)$;
therefore $S_2 \circ S_1 = S$.

b) $S(D)=S_2 \circ S_1(D)=S_2(S_1(D))=S_2(D)=P$ since $OC=\frac{\sqrt{2}}{2}OD$ and $(\overrightarrow{OD}\,;\overrightarrow{OP})=\frac{\pi}{4}\ (2\pi)$.

$S(C)=D$, $S(D)=P$ and L is the mid point of $[DC]$, then $S(L)=Q$, the mid point of $[DP]$.

4- a) $h(B)=S\circ S(B)=S(L)=Q$; $h(C)=S\circ S(C)=S(D)=P$.

b) $S=Sim(I\,;\frac{1}{2}\,;\frac{\pi}{2})$, then $h=S\circ S=Sim(I\,;\frac{1}{4}\,;\pi)$. Therefore h is the dilation $(I\,;-\frac{1}{4})$.

$h(B)=Q$, then $I\in(BQ)$; $h(C)=P$, then $I\in(CP)$. Therefore , I is the point of intersection of the two straight lines (BQ) and (CP).

c) $S(C)=D$, then $(\overrightarrow{IC}\,;\overrightarrow{ID})=\frac{\pi}{2}\ (2\pi)$; therefore , I belongs to the circle (γ) .

d) $S(L)=Q$, then $(\overrightarrow{IL}\,;\overrightarrow{IQ})=\frac{\pi}{2}\ (2\pi)$; therefore , (LI) is perpendicular to (BQ) at I ; therefore

(BQ) is the tangent to (γ) at I .