Exercise 1 [10 points] **Energy transfer and interaction**

A small marble (A), of mass m_A, is launched with the velocity $\vec{v}_A$ towards another small marble (B), of mass $m_B = \alpha\, m_A$, initially at rest. We suppose that the collision is elastic and that the velocities $\vec{v}'_A$ and $\vec{v}'_B$ of (A) and (B), respectively just after the collision, remain collinear to $\vec{v}_A$.

1. Determine, in terms of α and $\vec{v}_A$, the expressions of $\vec{v}'_A$ and $\vec{v}'_B$.

2.1. Express, in terms of α, the transfer factor $\eta = \frac{\text{KE(B)final}}{\text{KE(total initial of A and B)}}$, ratio of the kinetic energy transferred to (B) by the total initial kinetic energy of (A) and (B).

2.2. Identify the value of α for which this transfer is maximum and specify the corresponding situation.

3. The collision duration Δt is short so as to consider that $\frac{\Delta\vec{P}}{\Delta t} \approx \frac{d\vec{P}}{dt}$. Determine, during Δt:

3.1. the expression of the variation of the linear momentum $\Delta\vec{P}_A$ of (A) and that $\Delta\vec{P}_B$ of (B) in terms of m_A, α and $\vec{v}_A$.

3.2. the expression of the force $\vec{F}_{A/B}$ exerted by (A) on (B) in terms of Δt, m_A, α and $\vec{v}_A$.

Exercise 2 [17 points] **RC series circuit**

The electric circuit, shown in figure 1, is formed of an ideal generator of *e.m.f.* E=200V, two resistors (R_1) and (R_2) of respective resistances R_1 and R_2, a resistor (R) of adjustable resistance R, a capacitor (C) of capacitance C, initially uncharged, and a switch K.

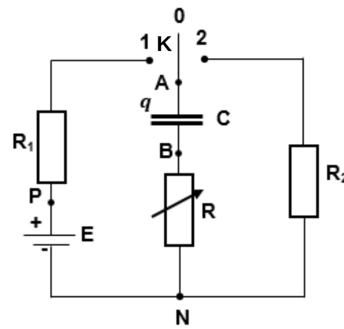


Fig. 1

A-I. Theoretical study of the charging

At the instant $t_0 = 0$, the switch K is placed in the position 1. At an instant t, the circuit carries a current i, $u_C = u_{AB}$ is the voltage across (C) that is charged by $q = Cu_C$.

1. Derive the differential equation that describes the variation of i as a function of t.

2. The solution of this equation being of the form $i = A.e^{\frac{-t}{\tau_1}}$, show that the expressions of A and τ_1 are $A = \frac{E}{R+R_1}$ and $\tau_1 = (R+R_1)\,C$.

3. Deduce the expression of u_c in terms of de E, t and τ_1.

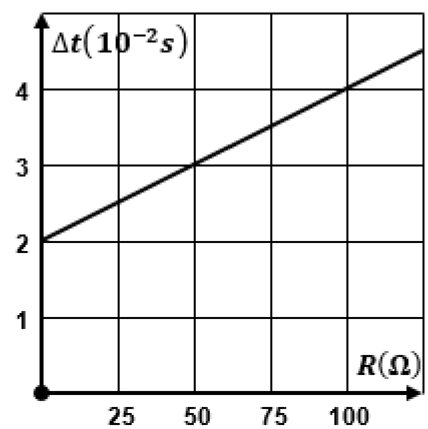


Fig. 2

A-II. Experimental study of the charging

<u>Experiment 1</u>: We intend to determine experimentally the value of C and that of R_1. To do that, we give R different values and we measure, for each of these values, the duration Δt at the end of which (C) is practically completely charged. A device (D) allowed us to obtain the plot of the curve giving Δt as a function of R (Fig. 2).

1. Justify theoretically the shape of the curve $\Delta t = f(R)$.

2. Referring to the graph, determine C and deduce R_1.

<u>Experiment 2</u>: During this experiment, we give R a constant value R_0 and by means of the device (D), the plot of the curve giving $\frac{di}{dt}$ as a function of i was obtained (Fig. 3).

1. Determine, using the graph, the value of τ_1.

2. Calculate R_0.

3. Determine the initial value i_0 of i.

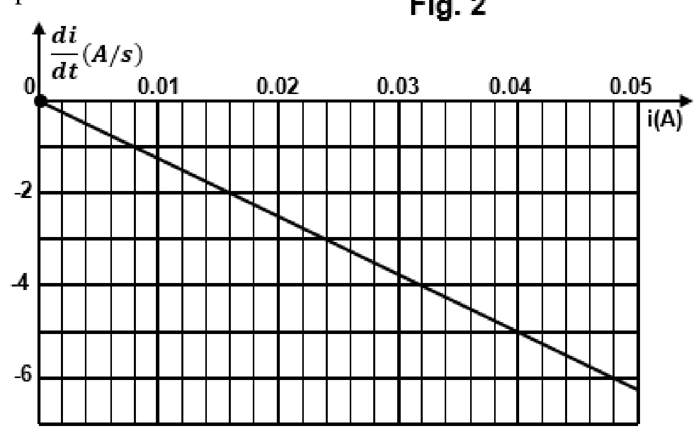


Fig. 3

B- Study of the discharging

When the current becomes nil in the circuit, the switch K is turned to position 2 at an instant chosen as a new origin of time ($t_0 = 0$); (C) starts to discharge through (R) and (R_2). At an instant t, the circuit carries a current i, in the real direction, and (C) is charged by $q = C.u_C = C.u_{AB}$.

1. Derive the differential equation that describes the variation of i as a function of t.

2. Verify that $i = \frac{E}{R+R_2} e^{-\frac{t}{\tau_2}}$ is the solution of the previous differential equation with $\tau_2 = (R + R_2)\,C$.

3. By means of (D), the plot of the curves giving the electric energy Ec stored by (C), as a function of time, was obtained for two different values of R.

$E_{C1} \rightarrow$ for $R = R_{01}$ and $E_{C2} \rightarrow$ for $R = R_{02}$.

Referring to the graphs of figure 4:

3.1. compare, with justification, the values of R_{01} and R_{02}.

3.2. calculate, between the instants $t_0 = 0$ and t_1, the energy dissipated by Joule's effect by the circuit for $R = R_{01}$ and that for $R = R_{02}$, and then, conclude.

3.3. compare the energy dissipated by Joule's effect by the circuit for $R = R_{01}$ to that for $R = R_{02}$ between the instants $t_0 = 0$ and t_2.

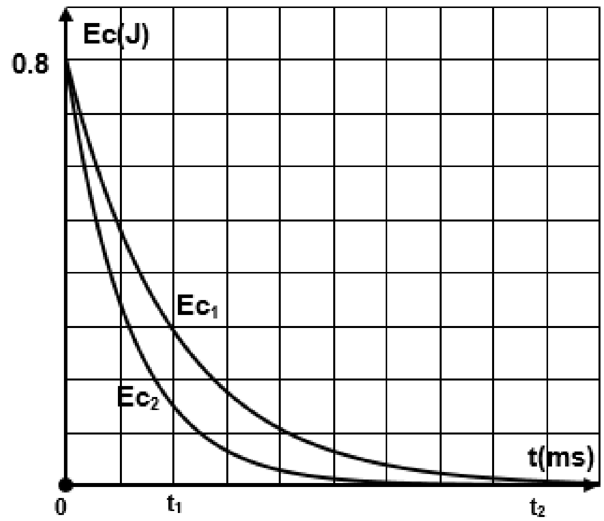
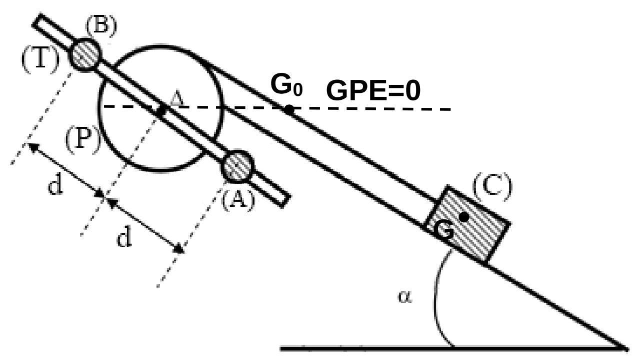


Fig. 4

Exercise 3 [15 points] Inclined plane and a system in rotational motion

The system (S), shown in the adjacent figure, is formed of a homogeneous rod (T) fixed on a pulley (P) of radius r = 0.20 m that can rotate about a horizontal (Δ) axis passing through its center. The moment of inertia of the set [(P), (T)] with respect to (Δ) is I_0. Two particles (A) and (B), each of mass m, that may slide on (T), are fixed at equal distance d from (Δ). The moment of inertia of the set [(T), (P), (A), (B)] is $I_\Delta = I_0 + 2md^2$.

An inextensible wire, of negligible mass, wounded around the groove of (P), is attached at the other end, to a particle (C), of mass M = 0.20 kg and of center of inertia G, that may slide along the line of greatest slope of an inclined plane that makes an angle $\alpha = 30^o$ with the horizontal.



Take:

- the wire does not slide on the groove of the pulley;
- the forces of friction are supposed negligible;
- the horizontal plane passing through G_0, position of G at the instant $t_0 = 0$, and the axis of rotation (Δ), is the reference level for the gravitational potential energy;
- $g = 10\ m/s^2$.

(S) is released from rest at the instant $t_0 = 0$. At an instant t, the abscissa of G is x, its velocity and acceleration are v and a respectively, the angular abscissa of (P) is θ, its angular velocity is θ' and its angular acceleration is θ''. We give d different values, and by means of an appropriate device, we measure, for each value of d, the value v_1 of v at the end of the distance $x_1 = 0.50$ m covered by G. The measurements are grouped in the table below:

d(m)	0	0.10	0.20	0.30	0.40
v_1(m/s)	1.49	1.41	1.24	1.05	0.89
a(m/s^2)		1.99		1.10	0.79
θ''(rad/s^2)	11.10			5.50	3.95
I_Δ(kg.m^2)		0.0121		0.0284	0.0426

1.1. Show that, at an instant t, the expression of the mechanical energy of the system [(S), Earth], is given by:

$ME = \frac{1}{2} I_\Delta \dot{\theta}^2 + \frac{1}{2} M r^2 \dot{\theta}^2 - M g r \theta \sin\alpha$

1.2. Using the system [(S), Earth], derive the expression of θ'' and deduce the nature of motion of the pulley.

2. Complete the preceding table of measurements.

3. Deduce I_0 and m.

4. We fix each particle at the distance d = 0.10 m. (C) is released from rest at the instant $t_0 = 0$. At the instant $t_1 = 5.0$ s, we cut the wire, and, starting from t_1, an instant chosen as a new origin of time, ($t_0 = 0$), (P) is subjected to a resistive couple. At an instant t, (P) is moving with an angular velocity θ' and the moment of the resistive couple is $\mathscr{M} = -K \cdot \theta'$, with $K = 1.10\times10^{-3}$ SI. (P) stops after covering 5 turns.

The horizontal plane passing through (Δ) is the reference level for the gravitational potential energy.

4.1. Derive the differential equation in θ', relative to the system [(P), (T), (A), (B), Earth], knowing that $\frac{dME}{dt} = \mathscr{M}\theta'$, where $\mathscr{M}$ is the sum of the moments of the external forces and couples applied on this system at the instant t.

4.2. The solution of this differential equation is of the form: $\theta' = a + b.e^{-\frac{t}{\tau}}$.

Determine the value of each of these constants a, b and τ.

4.3. Using the system [(P), (T), (A), (B), Earth], determine the duration at the end of which (P) stops.

Exercise 4 [18 points] Study of a ball motion

Consider a ball (B) of mass m = 200 g, taken as a particle, and a track formed of a rough rectilinear part (AB), of length AB = 4.5 m, inclined by an angle α with respect to the horizontal and a horizontal smooth rectilinear part (BC) (Fig. 1).

The ball is launched from A, at the instant $t_0 = 0$, with the velocity $\vec{V}_0$. It passes, at an instant t, by the point M where AM = ℓ. At B, we suppose that the speed keeps the same value V_B when (B) passes from (AB) to (BC).

When it reaches the point C with the velocity $\vec{V}_C$, at the altitude h = 3.2 m from the ground, the ball leaves the part (BC) thus undergoing a free fall towards the ground at D (Fig. 1).

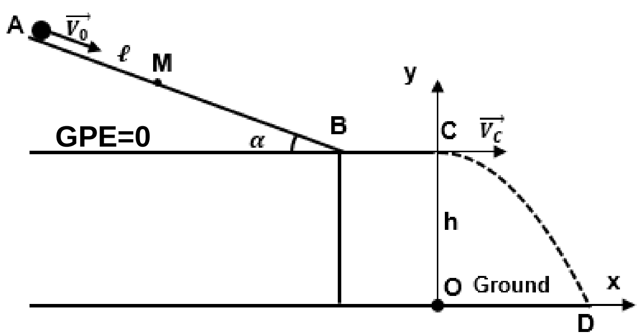


Fig. 1

Take:

- the horizontal plane passing through B as the reference level for the gravitational potential energy.
- $g = 10\ m/s^2$.

A- Motion of the ball between A and C

The curves of figure 2 show the mechanical energy ME and the gravitational potential energy GPE of the system (ball, Earth) as a function of ℓ.

Referring to (Fig. 1) and (Fig. 2):

1.1. show that, through a point M, the expression of GPE is:

GPE(M) = (9 – 2ℓ) sinα (GPE in J and ℓ in m).

1.2. deduce the value of the angle α.

2. determine V_0, and show that $V_B = 3.0$ m/s.

3. determine the value f of the force of friction $\vec{f}$, supposed constant, between A and B.

4. determine the speed of the ball when it reaches C.

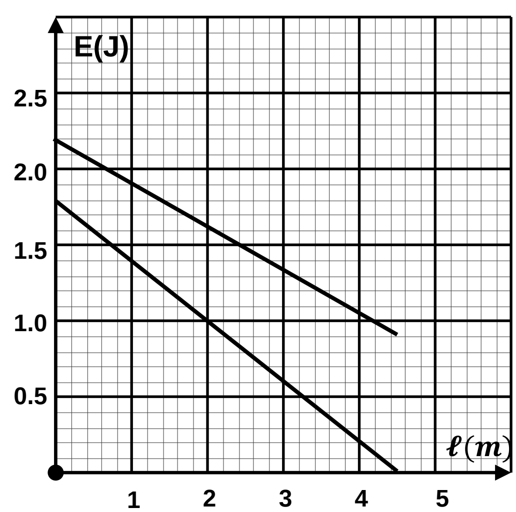


Fig. 2

B- Motion starting from C

When it leaves the point C, at an instant t_0 taken as a new origin of time, $t_0 = 0$, the ball undergoes a free fall in the vertical plane Oxy containing $\vec{V}_C$ of value $V_C = 3.0$ m/s. At an instant t, the coordinates of the ball are x and y.

1. Applying Newton's second law $\Sigma\vec{F} = \frac{d\vec{P}}{dt}$, determine, as a function of time t, the expression of the horizontal component $P_x = m.V_x$ and that of the vertical component $P_y = m.V_y$ of the linear momentum $\vec{P}$ of the ball.

2. Deduce that the time equations x and y of motion of the ball in the system of coordinates Oxy are respectively given by $x = 3\,t$ and $y = -5\,t^2 + 3.2$ (x and y in m and t in s).

3.1. Determine the duration taken by the ball to pass from C to D, just before reaching the hard ground.

3.2. Determine V_{yD}, the speed V_D and the direction of the velocity $\vec{V}_D$ of the ball at the point D.

C- Restitution coefficient and successive bounces

In order to determine the restitution coefficient of the collision of (B) with the ground, we measure the duration of each bounce.

At D, the ball bounces, just after the collision with the ground, with a velocity whose vertical component is written as $\vec{V}_{y1} = -q\ V_{yD}\ \vec{j}$ where q, is a constant, called the restitution coefficient ($0 < q < 1$).

We take the instant just after the collision as an origin of time, ($t_0 = 0$), and the impact point as an origin of space.

1.1. Justify that the expression, just after the collision, of the vertical component $P_y = m.V_y$ of the linear momentum $\vec{P}$ of (B) is $P_y = -\,mg\,t + mV_{y1}$.

1.2. Deduce that (B) reaches its maximum height at the end of the duration $\tau = \frac{V_{y1}}{g} = \frac{qV_{yD}}{g}$.

1.3. Justify that the duration Δt_1 of the first bounce of (B) is $\Delta t_1 = \frac{2\,V_{y1}}{g} = \frac{2qV_{yD}}{g}$.

2. The bounce occurs many times. Show that, after the n^{th} bounce, (B) goes up with a velocity whose vertical component is written as $V_{yn} = q^n.V_{yD} = q^{n-1}.V_{y1}$.

3. Deduce that the duration of the n^{th} bounce is written as $\Delta t_n = q^{n-1}\ \Delta t_1$.

4. The figure below (Fig. 3) shows $\ell n\ (\Delta t_n)$ as a function of n.

Determine q and Δt_1.

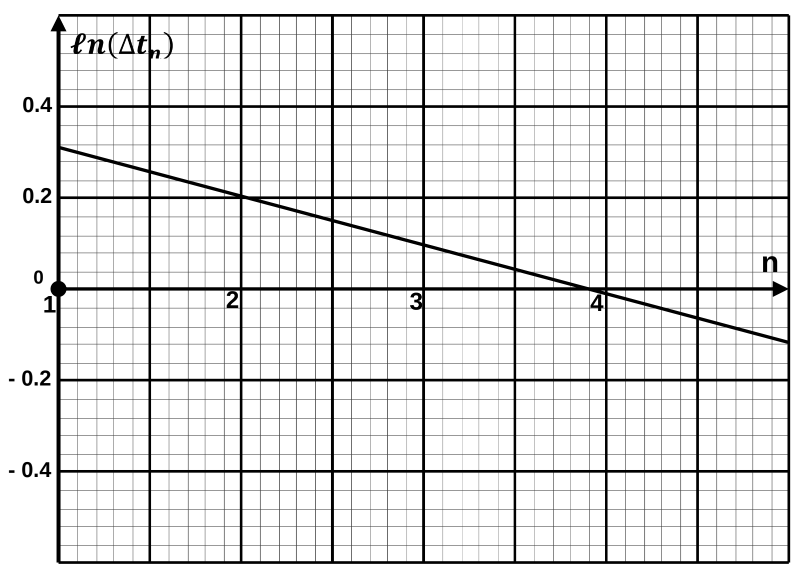


Fig. 3

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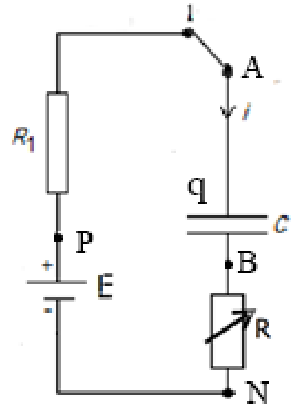
Exercise 1 (10 points)		
1.	Elastic collision, conservation of the linear momentum: $m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}'_A + m_B\vec{v}'_B$ $m_A V_A + 0 = m_A V'_A + \alpha\, m_A V'_B$ $V_A - V'_A = \alpha\, V'_B$ (1) Elastic collision, conservation of the kinetic energy $\frac{1}{2} m_A V_A^2 + 0 = \frac{1}{2} m_A V_A'^2 + \frac{1}{2} m_B V_B'^2$ $m_A V_A^2 + 0 = m_A V_A'^2 + \alpha\, m_A V_B'^2$ $V_A^2 - V_A'^2 = \alpha\, V_B'^2$ (2) $\frac{(2)}{(1)} = \frac{V_A^2 - V_A'^2}{V_A - V'_A} = \frac{\alpha\, V_A'^2}{\alpha\, V'_A}$ $V_A + V'_A = V'_B$ (3) (1)+ (3) $\Rightarrow 2\, V_A = (\alpha + 1)\, V'_B \Rightarrow V'_B = \frac{2V_A}{1+\alpha}$; Finally $\vec{v}'_B = \frac{2\vec{V}_A}{1+\alpha}$ (3) $\Rightarrow V'_A = V'_B - V_A = \frac{2V_A}{1+\alpha} - V_A = \frac{2V_A-(1+\alpha)V_A}{1+\alpha} = \frac{(1-\alpha)V_A}{1+\alpha}$; Finally $\vec{v}'_A = \frac{(1-\alpha)\vec{V}_A}{1+\alpha}$	**0.5** **0.5** **0.5** **1** **1**
2.1	$\eta = \frac{\frac{1}{2} m_B V_B'^2}{\frac{1}{2} m_A V_A^2 + 0} = \frac{\alpha m_A V_B'^2}{m_A V_A^2} = \frac{4.\alpha.V_A^2}{(1+\alpha)^2.V_A^2}$; then $\eta = \frac{4\alpha}{(1+\alpha)^2}$	**1**
2.2	The transfer coefficient η is maximum for $\frac{d\eta}{d\alpha} = 0$ $(1+\alpha)^2 - \alpha\times 2(1+\alpha) = 0 \Leftrightarrow 1 + 2\alpha + \alpha^2 - 2\alpha - 2\alpha^2 = 0 \Rightarrow 1 - \alpha^2 = 0$ So: $\alpha = \pm 1$ and since $\alpha > 0$, then $\alpha = 1$. ($\frac{d\eta}{d\alpha} > 0$, for $\alpha < 1$, $\frac{d\eta}{d\alpha} < 0$, for $\alpha > 1$). Identify the value of α for which this transfer is maximum. It is the case of a head-on collision, the marbles having the same mass: $V'_B = V_A$ and $V'_A = 0$, so with a total transfer of energy.	**1.5** **1**
3.1.	The variation of the linear momentum $\Delta\vec{P}_A$ of (A) is given by: $\Delta\vec{P}_A = \vec{P}'_A - \vec{P}_A = m_A[\frac{(1-\alpha)\vec{V}_A}{1+\alpha} - \vec{V}_A]$ and $\Delta\vec{P}_A = -\frac{(2\alpha\, m_A)}{1+\alpha}\vec{V}_A$ The variation of the linear momentum $\Delta\vec{P}_B$ of (B) is given by: $\Delta\vec{P}_B = \vec{P}'_B - \vec{P}_B = m_B\,\vec{V}'_B = \alpha\, m_A \frac{2\vec{V}_A}{1+\alpha}$ and $\Delta\vec{P}_B = \frac{(2\alpha\, m_A)}{1+\alpha}\vec{V}_A$	**1** **1**
3.2.	The forces other than the force $\vec{F}_{A/B}$ exerted by(A) on (B) compensate each other ; so, according to Newton's second law: $\vec{F}_{A/B} = \frac{d\vec{P}_B}{dt} \approx \frac{\Delta\vec{P}_B}{\Delta t} = \frac{(2\alpha\, m_A)\vec{V}_A}{(1+\alpha)\Delta t}$	**1**

Exercise 2 (17 points)		
A.I-1.	Law of addition of voltages $u_{PN} = u_{PA} + u_{AB} + u_{BN}$. with $u_{AB} = u_C$ and according to ohm's law: $u_{PA} = R_1\ i$ and $u_{BN} = R\ i$. $E = R_1\ i + u_C + R\ i = (R_1 + R)\ i + u_C\ \ \forall t$ i is directed towards the armature of charge q; So, $i = \frac{dq}{dt}$ & $q = C\ u_C$; Then $i = C\ \frac{du_C}{dt}$. Deriving with respect to time: $0 = (R_1 + R)\ \frac{di}{dt} + \frac{du_C}{dt} \Rightarrow (R_1 + R)\ \frac{di}{dt} + \frac{1}{C}\ i = 0$ We get $\frac{di}{dt} + \frac{1}{(R_1+R)C}\ i = 0\ \forall$ t. 	**0.5** **1**
A.I-2.	The solution is $i = Ae^{\frac{-t}{\tau_1}} \Rightarrow \frac{di}{dt} = -\frac{A}{\tau_1}e^{\frac{-t}{\tau_1}}$ At the instant $t_0 = 0$, $u_C = 0$ and $i_0 = A \Rightarrow E = (R_1 + R)\ i_0 = (R_1 + R)\ A$ & $A = \frac{E}{(R_1+R)}$ By replacing in the differential equation, we get: $-\frac{A}{\tau_1}e^{\frac{-t}{\tau_1}} + \frac{1}{(R_1+R)C}Ae^{\frac{-t}{\tau_1}} = 0 \Leftrightarrow Ae^{\frac{-t}{\tau_1}}\ (-\frac{1}{\tau_1} + \frac{1}{(R_1+R)C}) = 0$ is verified at any instant t, Then: $(-\frac{1}{\tau_1} + \frac{1}{(R_1+R)C}) = 0$ and $\tau_1 = (R_1 + R)\ C$	**1** **1**
A.I-3.	Knowing that $E = (R_1 + R)\ i + u_C$, then $u_C = E - (R_1 + R)i = E - (R_1 + R)\frac{E}{(R_1+R)}\ e^{\frac{-t}{\tau_1}}$ Thus, $u_C = E(1-e^{\frac{-t}{\tau_1}})$.	**0.5**
A.II (1st)-1.	Practically, $\Delta t = 5\ \tau_1$ = is the duration after which the capacitor becomes completely charged. $\Delta t = 5\ \tau_1 = 5\ (R_1 + R)\ C = 5C.R + 5\ R_1C$, is an equation of the form $\Delta t = a\ R + b$, Where $b = 5R_1C$ and $a = 5\ C$ is the slope of the straight carrying the curve.	**1**
A.II (1st)-2.	The slope $a = 5\ C = \frac{(4-2)\times10^{-2}}{100}$ and $C = 0.4\times10^{-4} = 4\times10^{-5}$ F or 40 μF. The constant b is written (ordinate at the origin), $b = 5\ R_1C = 2\times10^{-2}$ and $R_1 = \frac{2\times10^{-2}}{5\times40\times10^{-6}} = 100\ \Omega$.	**1** **1**
A.II (2nd)-1.	From the differential equation, we get $\frac{di}{dt} = -\frac{i}{\tau_1}$: $-\frac{1}{\tau_1}$ is then the slope p of the straight line carrying the curve. $p = -\frac{1}{\tau_1} = \frac{-5-0}{0.04-0} = -125$ and $\tau_1 = \frac{1}{125} = 0.008$ s.	**1** **0.5**
A.II (2nd)-2.	Knowing that $\tau_1 = (R_1 + R_0)\ C$, then $R_0 = \frac{\tau_1}{C} - R_1 = \frac{0,008}{40\times10^{-6}} - 100 = 100\ \Omega$.	**0.5**
A.II (2nd)-3.	At $t_0 = 0$, $u_{C0} = 0$, so: $E = (R_1 + R_0)\ i_0$, then: $i_0 = \frac{E}{(R_1+R_0)} = \frac{200}{(100+100)} = 1$ A.	**1**

Exercise 2 (17 points)		
B-1.	At the end of charging $u_C = E$. So, at the instant $t_0 = 0$, $u_{C0} = E$. At an instant t, i, is in the real direction, is directed towards the armature carrying the charge (-q). $i = -\frac{dq}{dt} = -C\frac{du_C}{dt}$ and $\frac{du_C}{dt} = -\frac{i}{C}$. Law of addition of voltages: $u_{AN} = u_{AB} + u_{BN}$ with $u_{AB} = u_C$ and according to Ohm's law: $u_{AN} = R_2\ i$ and $u_{BN} = -R\ i$. $R_2\ i = -R\ i + u_C \Rightarrow u_C = (R_2 + R)\ i$ Deriving with respect to time: $\frac{du_C}{dt} = (R_2 + R)\frac{di}{dt} = -\frac{i}{C}$ The differential equation is written: $\frac{di}{dt} + \frac{1}{(R_2+R)C}\ i = 0$	**1** **1**
B-2.	$i = \frac{E}{(R+R_2)} e^{\frac{-t}{\tau 2}}$; $\frac{di}{dt} = -\frac{E\ e^{\frac{-t}{\tau 2}}}{\tau_2(R+R_2)}$ Replacing in the differential equation: $-\frac{E\ e^{\frac{-t}{\tau 2}}}{\tau_2(R+R_2)} + \frac{1}{(R_2+R)C}\frac{E}{(R+R_2)} e^{\frac{-t}{\tau 2}} = \frac{E\ e^{\frac{-t}{\tau 2}}}{(R+R_2)}\left[-\frac{1}{\tau_2} + \frac{1}{(R_2+R)C}\right] = 0\ \ \forall t$ $\tau_2 = (R + R_2)C$	**1.5**
B-3.1.	$E_{C1} = ½\ C\ u_{c1}^2$; $u_{C1} = E\ e^{\frac{-t}{\tau 21}}$; $\tau_{21} = (R_2 + R_{01})\ C$ $E_{C2} = ½\ C\ u_{c2}^2$; $u_{C2} = E\ e^{\frac{-t}{\tau 22}}$; $\tau_{22} = (R_2 + R_{02})\ C$ R_{01} ?? R_{02} In the case R_{01}, the discharge takes more time that of the case R_{02}. So, since $\tau_{21} > \tau_{22}$, then $R_{01} > R_{02}$.	**1**
B-3.2.	For R_{01}: between $t_0 = 0$ and t_1 $E_{th1} = 0.8J – 0.3J = 0.5J$ For R_{02}: between $t_0 = 0$ and t_1 $E_{th2} = 0.8J – 0.15J = 0.65J$ $E_{th2} > E_{th1}$ withe $R_{01} > R_{02}$.	**0.5** **0.5** **0.5**
B-3.3.	For $t_0 = 0 \Rightarrow (E_{C1})_0 = (E_{C2})_0 = ½\ CE^2$. For t_2, we have a complete discharging, then $E_{th1} = E_{th2} = ½\ CE^2$	**1**

Exercise 3 (15 points)

1.1.

At the instant t, the mechanical energy of the system is: ME= ½ $I_\Delta \dot{\theta}^2$ + ½ Mv^2 – M g h

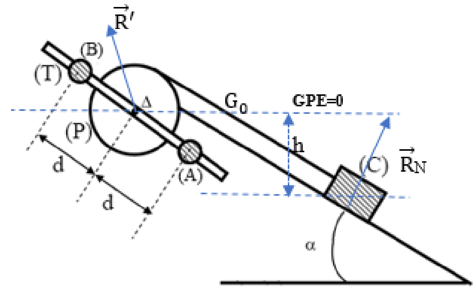
The wire does not slide, then x = r θ, v = r $\dot{\theta}$ and

a = r $\ddot{\theta}$. and also, h = x sinα.

0.5

1

1.2.



The initial mechanical energy of the system is nil : ME (0) = 0.

Due to the absence of dissipative forces, then the mechanical energy of the system is conserved.

Then: ½ $I_\Delta \dot{\theta}^2$ + ½ M $r^2 \dot{\theta}^2$ – M g r θ sinα = 0

and ½(I_Δ + M r^2) $\dot{\theta}^2$ = M g r θ sinα

The derivative with respect to time of the equation gives:

½(I_Δ + M r^2) 2 $\dot{\theta}\ddot{\theta}$ = M g r $\dot{\theta}$ sinα ; but $\dot{\theta} \neq 0$ during motion, then:

$\ddot{\theta} = \frac{M\,g\,r\,\sin\alpha}{I_\Delta + M\,r^2}$ = constant.

$\ddot{\theta} = \frac{0.2\times10\times0.2\times0.5}{I_\Delta + 0.2\times(0.2)^2} = \frac{0.2}{I_\Delta + 0.008}$ (rad/s^2)

The rotational motion of (P) is uniformly accelerated

2

2.

$a = \frac{v_1^2}{2x_1} = \frac{1.49^2}{2\times0.5} = 1.49^2 = 2.22$ m/s^2.

$\ddot{\theta} = \frac{a}{r} = 5$ a. $I_\Delta + 0.008 = \frac{0.2}{\theta''} \Rightarrow I_\Delta = \frac{0.2}{\theta''} - 0.008$

d(m)	0	0.10	0.20	0.30	0.40
v_1(m/s)	1.49	1.41	1.24	1.05	0.89
a(m/s^2)	**2.22**	1.99	**1.54**	1.10	0.79
θ"(rad/s^2)	11.10	**9.95**	**7.70**	5.50	3.95
I_Δ(kg.m^2)	**0.010**	0.0121	**0.0180**	0.0284	0.0426

2.5

3.

$I_\Delta = I_0 + 2\,md^2$; For d = 0, $(I_\Delta)_0 = I_0$ = 0.010 kg.m^2.

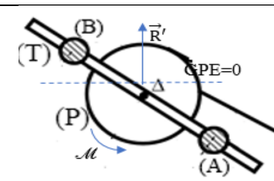
$(I_\Delta)_1 = I_0 + 2\,md^2$ = 0.010 + m×0.02 = 0.0121 kg.m^2

and m = $\frac{0.0021}{0.02}$ = 0.105 kg.

1

1

4.1.



System [(P), (T), (A), (B), Earth]: GPE = 0

and ME = KE = ½ $I_\Delta \theta'^2$;and $\frac{dME}{dt} = \mathcal{M}\theta' \Rightarrow \frac{dME}{dt} = I_\Delta \theta' \theta''$;

So: $I_\Delta \theta' \theta''$ = - K θ'·θ' (θ' ≠ 0 during motion)

We get, $\theta'' + \frac{K}{I_\Delta}\cdot\theta' = 0$

2.5

4.2.

$\theta' = a + b\,e^{-t/\tau} \Rightarrow \theta'' = -b/\tau\, e^{-t/\tau}$

$-b/\tau\, e^{-t/\tau} + \frac{K}{I_\Delta} a + \frac{K}{I_\Delta} b\, e^{-t/\tau} = 0 \Rightarrow \frac{K}{I_\Delta} a + b\, e^{-t/\tau}(-1/\tau + \frac{K}{I_\Delta}) = 0$

Identifying, we get: a = 0, $-\frac{K}{I_\Delta} = -\frac{1}{\tau}$ and $\tau = \frac{I_\Delta}{K} = \frac{0.0121}{1.10\times10^{-3}}$, so; τ = 11 s.

We have: d = 0.10 m ⇒ $I_\Delta = I_{\Delta1}$ = 0.0121 kg.m^2.

θ" = 9.95 rad/s^2 ; θ_0 = 0 so b= $\theta'_0 = \theta'_1$ = 49.75 rad/s

2

4.3.

The integration gives us: θ = -11×49.75 $e^{-t/11}$ + constant.

But, at t_0 = 0, θ=0 then constant=547.25

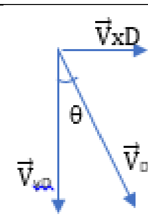
Thus, θ = 547.25 (1 – $e^{-t/11}$)

(P) stops: 5 turns ⇒ θ = 5× 2π = 10 π rad

So: 10 π = 547.25 (1 – $e^{-t/11}$);

$t = -11$ ln (0.9426)= 0.65 s

2.5

Exercise 4 (18 points)		
A-1.1.	The expression of the gravitational potential energy GPE of the system (ball, Earth) is given by: $GPE(M) = m\,g\,h_M$ where $h_M = MB.\sin\alpha = (AB - \ell)\sin\alpha$. and $h_A = (4.5 - \ell)\times\sin\alpha$. So, $GPE(M) = 0.2\times10(4.5 - \ell)\times\sin\alpha = (9 - 2\ell)\sin\alpha$ (GPE in J and ℓ in m).	**1.5**
A-1.2.	At the instant $t_0 = 0$, $\ell = 0$ and $GPE(0) = 1.8$ J. So, $1.8 = 9\sin\alpha$ and $\sin\alpha = 0.2$ then $\alpha = 11.5^o$.	**1.5**
A-2.	We have KE = ME –GPE. At A: (ℓ=0) KE(A) = 2.2 – 1.8 = 0.4 J. $KE(A) = ½\,m\,V_0^2 = ½\times0.2\times V_0^2 = 0.4$ and $V_0 = \sqrt{4} = 2.0$ m/s. At B: KE(B) = 0.9 – 0 = 0.9 J. $KE(B) = ½\,m\,V_B^2 = ½\times0.2\times V_B^2 = 0.9$ and $V_B = \sqrt{9} = 3.0$ m/s.	**1** **1**
A-3.	The variation in the mechanical energy between A and B is equal to the work done by the force of friction because the work done by the normal reaction is zero: $\Delta ME = W_{AB}(\vec{f})$ So, $W_{AB}(\vec{f})$ =ME(B) –ME(A) = 0.9 – 2.2 = - 1.3 J. We have $W_{AB}(\vec{f}) = -f.AB$, the value of the force of friction is $f = \frac{1.3}{4.5} = 0.29N$	**1.5**
A-4.	Between B and C, the mechanical energy of the system is conserved [part (BC) is smooth]. Since GPE = 0, then we have a conservation of kinetic energy, then the speed at B and C are equal $V_C = 3.0$ m/s.	**0.5**
B-1.	At $t_0 = 0$, $V_{0x} = V_C = 3.0$ m/s and $V_{0y} = 0$ $x_0 = 0$ and $y_0 = h = 3.2$ m The only force acting on (B) is its weight $\vec{w} = m\vec{g} = -m\,g\,\vec{j}$. Newton's second Law: $\Sigma\vec{F} = \frac{d\vec{P}}{dt} \Rightarrow \frac{d\vec{P}}{dt} = -m\,g\,\vec{j}$; by integration, The linear momentum $\vec{P} = -m\,g\,t\,\vec{j} + \vec{D}$. À $t_0 = 0$, $\vec{P}(0) = m\,\vec{V}_C = m\,V_C\,\vec{i}$ and $\vec{P} = -m\,g\,t\,\vec{j} + m\,V_C\,\vec{i}$. Then: $P_x = m\,V_C$ and $P_y = -m\,g\,t$	**1.5**
B-2.	We have $V_x = V_C$ = constant, and $V_x = \frac{dx}{dt} = V_C$ = constant, then $x = V_C\,t + C_x$. At $t_0 = 0$, $x = x_0 = C_x = 0$ and x = 3 t (x in m and t in s) We have $P_y = m\,V_y = -m\,g\,t$, then $V_y = -g\,t$. Since $V_y = \frac{dy}{dt} = -g\,t$, Then $y = -½\,g\,t^2 + C_y$. At $t_0 = 0$, $y_0 = h = C_y$; so $y = -½\,g\,t^2 + h$ and $y = -5\,t^2 + 3.2$ (y in m and t in s)	**1**
B-3.1.	At the point D, $y_D = 0$ and $-5t^2 + 3.2 = 0$ and $t = \sqrt{\frac{3.2}{5}} = +0.8s$	**1**
B-3.2.	$V_{yD} = -10\times0.8 = -8.0$ m/s. $V_D = \sqrt{V_x^2 + V_y^2} = \sqrt{3^2 + 8^2} = 8.54$ m/s. $\tan\theta = \frac{3}{8} = 0.375$ et $\theta = 20.6^o$. $\theta = \tan^{-1}(\frac{3}{8}) = 20.6^o$. 	**1.5**

C-1.1.	We have, according to Newton's second law, $\frac{d\vec{P}}{dt} = - m g \vec{j}$ $\Rightarrow \frac{dP_y}{dt} = - m g$ and $P_y = - m g t + C$. At $t_0 = 0$, $P_{y0} = mV_{y1} = C$ and $P_y = - m g t + m V_{y1}$. Then $V_y = - gt + V_{y1}$.	**1**
C-1.2.	The ball reaches its maximum height at the instant τ : $V_y = 0$, then $V_y = - g.\tau + V_{y1} = 0$; Thus, $\tau = \frac{V_{y1}}{g} = \frac{q\|V_{yD}\|}{g}$.	**0.5**
C-1.3.	Relying on the symmetry of motion, the duration of the upwards phase is equal to that of the fall and the duration of the first bounce Δt_1 is given by: $\Delta t_1 = 2\tau$ and $\Delta t_1 = \frac{2\,V_{y1}}{g} = \frac{2q\|V_{yD}\|}{g} = q\frac{2\|V_{yD}\|}{g}$	**0.5**
C-2.	During the second bounce, we get: $V_{y2} = q\,\|V_{y1}\|$ since q is a constant, then $V_{y2} = q^2\,\|V_{yD}\|$; We get $V_{yn} = q^n\,\|V_{yD}\| = q^{n-1}.\|V_{y1}\|$.	**1**
C-3.	The duration $\Delta t_1 = \frac{2\,V_{y1}}{g}$, the duration $\Delta t_2 = \frac{2\,V_{y2}}{g} = q\frac{2\|V_{y1}\|}{g} = q\,\Delta t_1$ ($\Delta t_3 = \frac{2\,V_{y3}}{g} = q\,\frac{2\|V_{y2}\|}{g} = q\,\Delta t_2 = q^2\Delta t_1$) and that of the nth bounce becomes: $\Delta t_n = \frac{2\,\|V_{yn}\|}{g} = q^{n-1}.\frac{2\,V_{y1}}{g}$. We get, $\Delta t_n = q^{n-1}\,\Delta t_1$which is verified for every value of n	**1**
C-4.	Since $\Delta t_n = q^{n-1}\,\Delta t_1$, then $\ell n\,(\Delta t_n) = (n-1)\,\ell nq + \ell n(\Delta t_1) = n\,\ell n(q) - \ell n(q) + \ell n\,(\Delta t_1)$. Thus, $\ell n(q)$ is the slope of the straight line carrying the curve: $\ell n(q) = \frac{-0.31}{2.9} = - 0.107$ and $q = 0.90$ For n = 1, $\ell n\,(\Delta t_n) = \ell n\,(\Delta t_1) = 0.31$ et $\Delta t_1 = 1.36$ s.	**2**