UNIVERSITE LIBANAISE
FACULTE DE GENIE

الجامعة اللبنانية
كلية الهندسة

Entrance exam 2016-2017 **July 2016**

Physics **Duration 2**

Exercise I: [12 pts] Interference

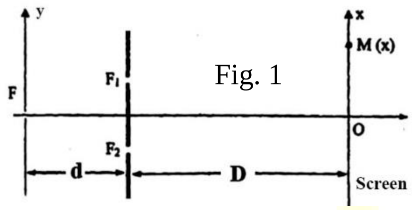

Fig. 1

Young's double slit experiment is performed in air (Fig. 1). These very fine slits, apart by $F_1F_2 = a = 1$ mm, are illuminated by a monochromatic light of wavelength in air $\lambda = 589$ nm, emitted by a fine source F placed at a distance d = 20 cm from the plane of the two slits. The source F and the two slits are horizontal. The observation is made on a screen located at a distance D = 100 cm from the plane of the two slits.

1. The optical path difference δ at a point M of abscissa x is given by: $\delta = F_2M - F_1M = \frac{ax}{D}$.

a) M being the center of a bright fringe, determine the expression of the interfringe distance i and calculate its value.

b) Specify the nature and the order of interference of the fringe whose center N is at 5.6 mm from O.

2. The source F is moved by a distance z along the Fy axis. We notice that the center of the central fringe moves up and takes the place of the center of the tenth dark fringe. Specify the displacement direction of F and calculate this displacement.

3. F is returned to its initial position and we insert, in front of F_2, a small parallel plate of index n = 1.5 and of thickness e. We notice that the interference pattern is displaced by b = 10 mm.

Determine the value of e, knowing that the optical path difference at M becomes: $\delta = \frac{ax}{D} + e(n-1)$.

Exercise II: [24 pts] About Energy

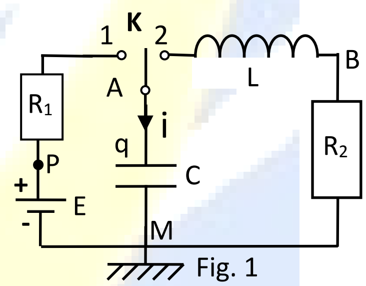

Fig. 1

The circuit of figure 1 is carried out with the following components: a generator of emf E = 4.0 V, two resistors (R_1) and (R_2) respectively of resistance $R_1 = 1$ kΩ and $R_2 = 400$ Ω, a coil of inductance L = 0.40 H and of negligible resistance, a capacitor of capacitance $C = 1.0\times10^{-6}$ F and a switch K.

A- Charging the capacitor

The capacitor is initially discharged and the switch K is moved to position 1 at the instant $t_0 = 0$. At an instant t, the circuit carries a current i and the voltage across the capacitor is $u = u_{AM} = E(1-e^{-\frac{t}{\tau}})$, where $\tau = R_1C$.

1. a) Calculate the value of τ and determine, as a function of time t, the expression of i.

b) Draw, as a function of time t, the shape of the variation of u and that of $u_{PA} = R_1$ i.

2 a) Show that, at an instant t, the power dissipated (by Joule's effect) by the circuit is: $P_J = \frac{E^2}{R_1}e^{-2\frac{t}{\tau}}$.

b) Deduce the energy W_J dissipated by the circuit during the charging mode and calculate its value.

c) Calculate the energy W_0 stored by the capacitor at the end of the charging mode.

d) Deduce the energy supplied by the generator to perform the charging of the capacitor.

B- Discharging the capacitor

The capacitor is fully charged. We move, at an instant $t_0 = 0$, the switch K to position 2.
Using an oscilloscope, we display, as a function of time, the voltages u_{AM} (curve 1) and u_{BM} (curve 2) (Fig. 2).

1. Energy Study

The voltages displayed (Fig.2) show that the circuit is the seat of pseudo-periodic oscillations of pseudo-period T.

a) Calculate, at the instant $t_0 = 0$, the energy W_1 stored by the LC circuit.

b) Determine, at the instant t_1, when the two curves intersect for the first time:

i) the current i.

ii) the electromagnetic energy W_2 stored by the LC circuit.

c) Calculate the energy dissipated by (R_2) between the instants $t_0 = 0$ and t_1.

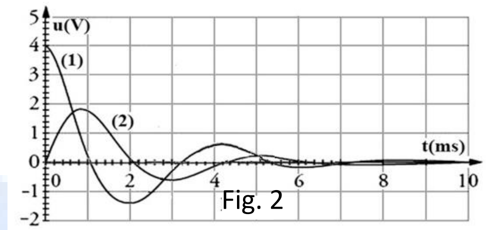

Fig. 2

2. Electrical Oscillations

a) Show that the differential equation that describes the variation of the voltage u is written as: $\ddot{u} + 2\lambda\dot{u} + \omega_0^2 u = 0$, where $\lambda = \frac{R_2}{2L}$ and $\omega_0 = \sqrt{\frac{1}{LC}}$.

b) The solution of this differential equation is of the form: $u = B\,e^{-\lambda t}\cos(\omega t - \phi)$, where ω, B and ϕ are constants.

i) Take $\delta = \ln\left(\frac{u(t)}{u(t+T)}\right)$, with $T = \frac{2\pi}{\omega}$. Show that $\delta = \lambda \cdot T$.

ii) Referring to the curve (1) of Figure 2, determine the corresponding values of T, δ, λ, B and ϕ.

Exercise III: [24 pts] Swing

A swing, on which sits a child of mass m and of center of inertia G, is considered as a simple pendulum (S) of adjustable length $\ell = OG$ and of mass m at G. (S) can oscillate in a vertical plane about a horizontal axis (Δ) passing through O. We assume the forces of friction negligible and the oscillations of small amplitude.

At an instant t, the position of (S) is identified by its angular elongation θ relative to the vertical, $\dot{\theta} = \frac{d\theta}{dt}$ being its angular velocity. The horizontal plane passing through O is the reference level for the gravitational potential energy. Take $g = 9.8\ m/s^2$, $\cos\theta = 1 - \frac{\theta^2}{2}$ and $\sin\theta = \theta$ (θ in rad).

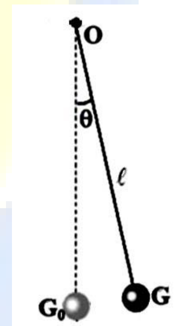

Fig. 1

A- Equation of motion

(S) is shifted in the positive direction from the equilibrium position, by a small angle θ_0, then left without speed at the instant $t_0 = 0$. (S) starts to oscillate with an amplitude θ_m supposed small.

1. a) Show that, at the instant t, the mechanical energy of the system [(S), Earth] is written: $ME = \frac{1}{2} m\ell^2\dot{\theta}^2 - mg\ell\cos\theta$.

b) Derive the differential equation in θ that describes the motion of (S).

c) Deduce the expression of the natural period T_0 of oscillations of (S) in terms of ℓ and g.

d) Show that the expression of the instant τ when the elongation of (S) reaches, for the first time, the value θ = 0 rad is given by: $\tau = \frac{\pi}{2}\sqrt{\frac{\ell}{g}}$.

2. a) The solution of the differential equation is of the form: $\theta = \theta_m \cos(\frac{2\pi}{T_0} t + \varphi)$. Determine the expression of θ_m and calculate the value of φ.

b) Show that the expression of the amplitude $\dot{\theta}_m$ of the angular velocity is given by: $\dot{\theta}_m = \sqrt{\frac{g}{\ell}}\,\theta_m$.

c) Calculate, for $\ell = 2.0$ m and $\theta_0 = 0.05$ rad, the values of θ_m, $\dot{\theta}_m$ and T_0.

B- Motion amplification

The child wishes, now, to increase (amplify) the amplitude θ_m of the oscillations starting from the instant $t_0 = 0$ where $\theta_0 = 0.05$ rad and $\dot{\theta}_0 = 0$.

- During the first passage by $\theta = 0$, at the instant t_1, the child instantaneously stands up (Fig. 2), thus moving up his center of inertia G from G_1 to G_2, with $\ell_2 = OG_2$.
- When passing by $\theta = -\theta_{1m}$ (maximum deviation in the negative direction), at the instant t_2 (Figure 3), the child sits down again instantaneously, lowering his center of inertia from G_2 to G_1.
- When passing again by $\theta = 0$ at the instant t_3, the child stands up again instantaneously, shifting his center of inertia from G_1 to G_2 (Fig. 2).
- When passing by $\theta = +\theta_{2m}$ (maximum deviation in the positive direction) at the instant t_4, the child sits down again instantaneously (Fig. 4), lowering his center of inertia G_2 to G_1.

1. Show that the expression of the period T of the oscillations of (S) is given by: $T = \pi(\frac{\sqrt{\ell_1}+\sqrt{\ell_2}}{\sqrt{g}})$.

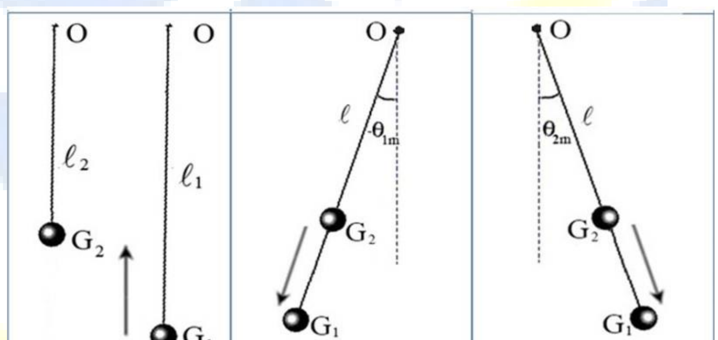

Fig 2 passage by $\theta=0$ Fig 3 passage by $\theta=\theta_{1m}$ Fig 4 passage by $\theta=\theta_{2m}$

2. Just before passing by $\theta = 0$, at the instant t_1^-, the child begins to rise, which means that $r(t_1^-) = \ell_1$ and just after, at the instant t_1^+, the child completes its rise, which means that $r(t_1^+) = \ell_2$.

a) Consider the physical quantity $K(t) = mr^2\dot{\theta}$ where r = OG, r being equal to ℓ_1 or ℓ_2. What does K(t) represent? Explain why $K(t_1^-) = K(t_1^+)$.

b) Let $\dot{\theta}_0^-$ and $\dot{\theta}_0^+$ be respectively the angular velocities of (S) at the instants (t_1^-) and (t_1^+). Determine, in terms of ℓ_1 and ℓ_2, the expression of the ratio $\dot{\theta}_0^+/\dot{\theta}_0^-$.

c) Deduce that $\theta_{2m}/\theta_{0m} = \ell_1^3/\ell_2^3$, where θ_{0m} and θ_{2m} are respectively the angular amplitudes of (S) at the instants t_0 and t = T.

3. a) After n oscillations, the amplitude is written as θ_{2nm}. Determine the expression of $\theta_{2nm}/\theta_{0nm}$ in terms of ℓ_1 and ℓ_2.

b) Calculate n so that $\theta_{2nm} = 10^o$, with $\ell_1 = 2.0$ m, $\ell_2 = 1.8$ m and $\theta_{0m} = 0.05$ rad.

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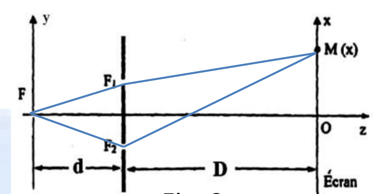
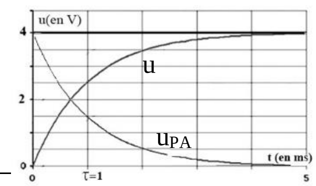
Exercise I : [12 pts] Interference

Q		Notes
1.a)	M is the center of a bright fringe. So $\delta = k\lambda$ with k an integer $\Rightarrow k\lambda = \frac{ax}{D} \Rightarrow$ For the fringe of order k: $x_k = \frac{k\lambda D}{a}$. For the fringe of order k-1: $x_{k-1} = \frac{(k-1)\lambda D}{a}$. The interfringe distance is: $i = x_k - x_{k-1} = \frac{\lambda D}{a}$. $i = \frac{589\times10^{-9}\times1.0}{1.0\times10^{-3}} =$ 0.589×10^{-3} m or 0.589 mm.  Fig. 2	2.5
b)	$\frac{x}{i} = \frac{5.6}{0.589} = 9.5 \Rightarrow x = 9.5\ i$, thus N is the center of tenth dark fringe and its order is k = 9.	2.5
2.	F has been displaced downward. The new position O' of the center of the central fringe is: x = 9.5 i = 5.6 mm. $FF_2 + F_2O' = FF_1 + F_1O' \Rightarrow F_2O' - F_1O' = FF_1 - FF_2$. Since $F_2O' > F_1O' \Rightarrow FF_1 > FF_2 \Rightarrow$ then the source F is displaced downward. Also, $\frac{ax}{D} = \frac{az}{d} \Rightarrow z = \frac{xd}{D} = \frac{5.6\times10^{-3}\times0.2}{1,0} = 1.12\times10^{-3}$ m or 1.12 mm.	3.5
3.	For the center of the central fringe, the optical path difference is zero. $\delta = \frac{ax}{D} + e(n-1) = 0 \Leftrightarrow e(n-1) = \frac{-a\,x}{D}$. So x < 0 and x = -b; thus the center of the central fringe is displaced downward. $\Rightarrow e(n-1) = \frac{ab}{D} \Rightarrow e(1.5-1) = \frac{1\times10^{-3}\times10\times10^{-3}}{1.0}$. So $e = 20\times10^{-6}$ m or 20 μm.	3.5

Exercise II : [24 pts] About Energy

Q		Notes
A-1. a)	As $\tau = R_1C$, then $\tau = 10^3\times10^{-6} = 10^{-3}$ s or 1 ms. At the instant t, $i = C\frac{du}{dt} = C\frac{E}{\tau}\,e^{-\frac{t}{\tau}} \Rightarrow i = \frac{E}{R_1}\,e^{-\frac{t}{\tau}}$ 	2
b)	See figure	2
2.a)	As the power dissipated by Joule effect is: $P_J = R_1i^2$, then: $P_J = \frac{E^2}{R_1}e^{-2\frac{t}{\tau}}$.	0.5
b)	The energy dissipated by the circuit during the charging: $W = \int_0^\infty P_J\,dt$, $W = \int_0^\infty \frac{E^2}{R_1}\,e^{-\frac{2t}{\tau}}\,dt = \frac{E^2}{R_1}[-\ \frac{\tau}{2}\ e^{-\frac{2t}{\tau}}]_0^\infty = \frac{E^2}{R_1}\times\frac{\tau}{2} = ½\ CE^2 = ½\ 1\times10^{-6}\ 4^2 = 8\times10^{-6}$ J.	2.5
c)	The energy stored by the capacitor is given by: $W_0 = ½\ CE^2$, since at the end of the charge u = E = 4 V. thus : $W_0 = ½\ 10^{-6}\times4^2 = 8\times10^{-6}$ J.	1
d)	$W = W_J + W_0 = 16\times10^{-6}$ J.	1
B-1. a)	At the instant $t_0 = 0$, the energy W_1 stored by the LC circuit is: $W_1 = ½\ Cu^2 + ½\ Li^2 = ½\ CE^2$, since at $t_0 = 0$, $u_0 = E$ and $i_0 = 0$. $W_1 = 8\times10^{-6}$ J.	1
b) i)	At the intersection, $u_2 = 1.7$ V. The current i: $i = u_2/R_2 = 1.7/4.0\times10^2 = 4.25\times10^{-3}$ A.	1.5
ii)	The electromagnetic energy W_2 stored : $W_2 = ½\ Cu^2 + ½\ Li^2 = ½\times10^{-6}\times1.7^2 + ½\times0.4\times(4.25\times10^{-3})^2 = 1.45\times10^{-6} + 3.61\times10^{-6} = 5.06\times10^{-6}$ J.	2.5
c)	Initially $W_1 = 8{,}0.10^{-6}$ J. When the curves intersect: $W_2 = 5.06\cdot10^{-6}$ J The energy dissipated by the resistor is $W_{dissip} = W_1 - W_2 = 2.94\times10^{-6}$ J.	1
2.a)	According to the law of addition of voltages: $u_{AM} = u_{AB} + u_{BM}$, where $u_{AM} = u$, $u_{AB} = -\,L\frac{di}{dt}$ and $u_{BM} = -R_2i$, with $i = \frac{dq}{dt}$ and $q = C\,u$. thus, $i = C\,\frac{du}{dt} \Rightarrow u_{AB} = -\,LC\frac{d^2u}{dt^2}$ and $u_{BM} = -R_2C\frac{du}{dt}$. thus : $u = -\,LC\frac{d^2u}{dt^2} - R_2C\,\frac{du}{dt} \Rightarrow \frac{d^2u}{dt^2} + \frac{R_2}{L}\,\frac{du}{dt} + \frac{1}{LC}\,u = 0 \Leftrightarrow \ddot{u} + 2\lambda\dot{u} + \omega_0^2u = 0$, where $\lambda = \frac{R_2}{2L}$ and $\omega_0 = \sqrt{\frac{1}{LC}}$.	2
b) i)	$\delta = \ln\left(\frac{u(t)}{u(t+T)}\right) = \ln\left(\frac{B\,e^{-\lambda t}\cos(\omega t-\phi)}{B\,e^{-\lambda(t+T)}\cos(\omega(t+T))-\phi)}\right) = \ln(e^{\lambda T})$. thus, $\delta = \lambda\cdot T$.	1.5
ii)	We have T = 4.2 ms. $\delta = \ln\left(\frac{4}{0.6}\right)$ gives: $\delta = 1.99$, as a result : $\lambda = \frac{\delta}{T} = \frac{1.99}{4.2\times10^{-3}} = 474\ s^{-1}$. At $t_0 = 0$, u = 4 V and $u_R = R_2i = 0 \Rightarrow i = 0$. $u = B\,e^{-\lambda t}\cos(\omega t-\phi)$, at $t_0 = 0$, $u = B\cos(\phi) = 4 \Rightarrow \cos(\phi) > 0$. $i = C\frac{du}{dt} = CB[-\lambda e^{-\lambda t}\cos(\omega t-\phi) - \omega e^{-\lambda t}\sin(\omega t-\phi)]$, at $t_0 = 0$, $i = -\,CB[\lambda\cos\phi - \omega\sin(\phi)] = 0$. $\tan\phi = \frac{\lambda}{\omega} = \frac{\lambda\cdot T}{2\pi} = \frac{1.99}{2\pi} = 0.316 \Rightarrow \phi = 0.306$ rad. $B\cos(\phi) = 4 \Rightarrow B = \frac{4}{\cos(0.306)} = 4.2$ V.	2.5 3

Exercise III : [24 pts] Swing

Q		Notes
A-1. a)	The mechanical energy, $E_m = ½\, I\cdot\dot{\theta}^2 + mgz \Rightarrow E_m = ½\, m\ell^2\dot{\theta}^2 + mgz$. As $z = -\ell\cos\theta$, then, $E_m = ½\, m\ell^2\dot{\theta}^2 - mg\ell\cos\theta$.	2
b)	No friction, therefore conservation of the mechanical energy. $ME = ½\, m\ell^2\dot{\theta}^2 + mgz = ½\, m\ell^2\dot{\theta}^2 - mg\ell\cos\theta$ = constant. The derivative with respect to time gives: $m\ell^2\, \dot{\theta}\ddot{\theta} + mg\ell\sin\theta\dot{\theta} = 0 \Rightarrow \ell\, \ddot{\theta} + g\sin\theta = 0$ since $\dot{\theta}$ is not always zero. For small θ, $\ddot{\theta} + \frac{g}{\ell}\theta = 0$.	2.5
c)	The general form of this differential equation is: $\ddot{\theta} + \omega_0^2\, \theta = 0$, we thus have simple harmonic oscillations of proper angular frequency ω_0. By comparison, $\omega_0^2 = \frac{g}{\ell}$. As $\omega_0 = \frac{2\pi}{T_0}$, giving: $T_0 = 2\pi\sqrt{\frac{\ell}{g}}$.	1.5
d)	The instant τ to which (S) reaches, for the first time, elongation $\theta = 0$ rad is $\tau = T_0/4 = \frac{\pi}{2}\sqrt{\frac{\ell}{g}}$.	0.5
2.a)	The angular velocity is expressed by: $\dot{\theta} = -\frac{2\pi}{T_0}\theta_m \sin(\frac{2\pi}{T_0}t + \varphi)$. At $t_0 = 0$, $\dot{\theta}_0 = -\frac{2\pi}{T_0}\theta_m \sin(\varphi) = 0 \Rightarrow \varphi = 0$ or π rad. At $t_0 = 0$, $\theta = \theta_m \cos(\varphi) > 0 \Rightarrow \varphi = 0$ rad and $\theta_m = \theta_0$. Thus: $\theta = \theta_m \cos(\frac{2\pi}{T_0}t)$.	2
b)	The expression of the angular velocity $\dot{\theta}$ is then given by: $\dot{\theta} = -\frac{2\pi}{T_0}\theta_m \sin(\frac{2\pi}{T_0}t)$. The amplitude of $\dot{\theta}$ is then: $\dot{\theta}_m = \frac{2\pi}{T_0}\theta_m = \sqrt{\frac{g}{\ell}}\theta_m$.	1
c)	For $\ell = 2.0$ m and $\theta_0 = 0.05$ rad : $\theta_m = 0{,}05$ rad ; $\dot{\theta}_m = \sqrt{\frac{9.8}{2}} \times 0.05 = 0.111$ rad/s and $T_0 = 2.84$ s.	1.5
B-1.	For $\ell = \ell_1$, $T_1 = 2\pi\sqrt{\frac{\ell_1}{g}}$ and for $\ell = \ell_2$, $T_2 = 2\pi\sqrt{\frac{\ell_2}{g}}$. For $\ell = \ell_1$, $t_1 = \frac{\pi}{2}\sqrt{\frac{\ell_1}{g}}$ and from t_1 to t_2 this takes ¼ of T_2. Thus: $t_2 = t_1 + \frac{\pi}{2}\sqrt{\frac{\ell_2}{g}} = \frac{\pi}{2}\sqrt{\frac{\ell_1}{g}} + \frac{\pi}{2}\sqrt{\frac{\ell_2}{g}} = \frac{\pi}{2}(\sqrt{\frac{\ell_1}{g}} + \sqrt{\frac{\ell_2}{g}})$. As a result, the expression of the period T of the oscillations of (S) is given by: $T = 2\, t_2 = \pi(\frac{\sqrt{\ell_1}+\sqrt{\ell_2}}{\sqrt{g}})$.	2.5
2. a)	The physical quantity $K(t) = mr^2\dot{\theta}$ is the angular momentum of (S) relative to (Δ) at an instant t. Between the instants t_1^- and t_1^+, (S) is subjected to the weight $\overrightarrow{W}$, a vertical force passing through O and the reaction of the axis (Δ) at O. So Σmoments/(Δ) = 0. Thus, we have the conservation of the angular momentum and as a result: $K(t_1^-) = K(t_1^+)$.	2.5

b)	$K(t_1^-) = m\ell_1^2\,\dot{\theta}_0^-$ and $K(t_1^+) = m\ell_2^2\,\dot{\theta}_0^+$. $K(t_1^-) = K(t_1^+) \Rightarrow m\ell_1^2\,\dot{\theta}_0^- = m\ell_2^2\,\dot{\theta}_0^+$. As a result: $\dot{\theta}_0^+/\dot{\theta}_0^- = \ell_1^2/\ell_2^2$.	2
c)	The amplitude of the angular speed is given by: $\dot{\theta}_0^- = \sqrt{\frac{g}{\ell_1}}\;\theta_{0m}$ and $\dot{\theta}_0^+ = \sqrt{\frac{g}{\ell_2}}\,\theta_{1m}$. With $\dot{\theta}_0^+/\dot{\theta}_0^- = \ell_1^2/\ell_2^2$ and $\dot{\theta}_0^+/\dot{\theta}_0^- = \sqrt{\frac{g}{\ell_2}}\,\theta_{1m}/\sqrt{\frac{g}{\ell_1}}\;\theta_{0m} = \sqrt{\frac{\ell_1}{\ell_2}}\,\theta_{1m}/\theta_{0m}$. Thus: $\ell_1^2/\ell_2^2 = \sqrt{\frac{\ell_1}{\ell_2}}\,\theta_{1m}/\;\theta_{0m}$. As a result : $\theta_{1m}/\;\theta_{0m} = \sqrt{\frac{\ell_2}{\ell_1}}\;\ell_1^2/\ell_2^2$ Lowering down in the positive direction: $\theta_{2m}/\;\theta_{1m} = \sqrt{\frac{\ell_2}{\ell_1}}\;\ell_1^2/\ell_2^2$, $\theta_{2m}/\theta_{0m} = \sqrt{\frac{\ell_2}{\ell_1}}\;\ell_1^2/\ell_2^2 \times \sqrt{\frac{\ell_2}{\ell_1}}\;\ell_1^2/\ell_2^2 = \ell_1^3/\ell_2^3$.	3.5
3.a)	$\theta_{2nm}/\theta_{0m} = (\ell_1^3/\ell_2^3)^n = \ell_1^{3n}/\ell_2^{3n}$	1
	$\theta_{2nm}/\theta_{0m} = (2^3/1.8^3)^n = 1.11^{3n} = 0.1745/0.05 = 3.49 \Rightarrow 3\,n\,\ln(1.11) = \ln(3.49) \Rightarrow n = 3.99 \approx 4$.	1.5