LEBANESE UNIVERSITY

FACULTY OF ENGINEERING

الجامعة اللبنانية
كلية الهندسة

Entrance exam 2013-2014 **PHYSICS** **Duration: 2 h**

14/7/2013

Exercise I [20 pts]: Forced oscillations. Resonance phenomena.

An elastic horizontal pendulum consists of a solid (S), of mass **m = 200 g,** attached to the end of a spring (R) of negligible mass and of stiffness **k**, the other end of the spring being fixed. The centre of inertia G of the solid can move on a horizontal axis (O, $\vec{i}$), O being the position of G when (S) is at equilibrium. A suitable device exerts on the pendulum an exciting force of adjustable angular frequency ω. G starts to oscillate on both sides of O. At an instant t, the abscissa of G is x and its velocity is $\vec{v} = v\vec{i} = \frac{dx}{dt}\vec{i}$; the exciting force is then of the form $\vec{F} = F\,\vec{i} = F_0\sin(\omega t+\phi)\,\vec{i}$, of constant amplitude F_0 and the solid (S) is subjected to a friction force of the form $\vec{f} = -\mathbf{h}\vec{v} = -h\,v\,\vec{i}$ where **h** is a positive constant.

1) Show that the differential equation in x associated with the motion of the pendulum is of the form:

$$\frac{d^2x}{dt^2}+\frac{h}{m}\frac{dx}{dt}+\frac{k}{m}x=\frac{F_0}{m}\sin(\omega t+\varphi).$$

2) In steady state, the solution of this differential equation is written as: $x = X_m\sin(\omega t)$.

a) Deduce, by giving ωt two particular values, the expression of tanϕ in terms of the data and show that the amplitude X_m is given by: $X_m = \dfrac{F_0}{\sqrt{h^2\omega^2+(k-m\omega^2)^2}}$.

b) By giving ω different values and by measuring, for each of these values, the corresponding value of X_m, we notice that we obtain an amplitude resonance phenomenon.

i) Determine the expression of the corresponding resonance angular frequency $\boldsymbol{\omega_r}$ in terms of the data.

ii) Draw the shape of the amplitude resonance curve for two different values of h.

3. a) Determine the expression of v.

b) Deduce the expression of the amplitude V_m of v in terms of the data.

c) i) Determine the resonance angular frequency $\boldsymbol{\omega_0}$ of V_m.

ii) Draw the shape of the resonance curve of the amplitude V_m for two different values of h.

4. We give each of the quantities ω, h and k a particular value. Using an appropriate device, we perform, in steady state, the recording of F and x; the curves (1) and (2) represent respectively the variation of F and x as a function of time, **where a vertical division represents a force of 0.3 N and a displacement of 1 cm and a horizontal division represents 30 ms.**

Referring to the curves:

a) Determine the expressions of F and x;

b) Determine the values of k and h.

Exercise II [22 pts]: Different roles of a coil

A. Opening and closing of the injector of a car

An electromagnet, formed of a coil, is used to control the opening and closing of the injector in the modern car engines. This coil can also be used as a metal detector. In this exercise, we are interested to determine the inductance L of the coil.

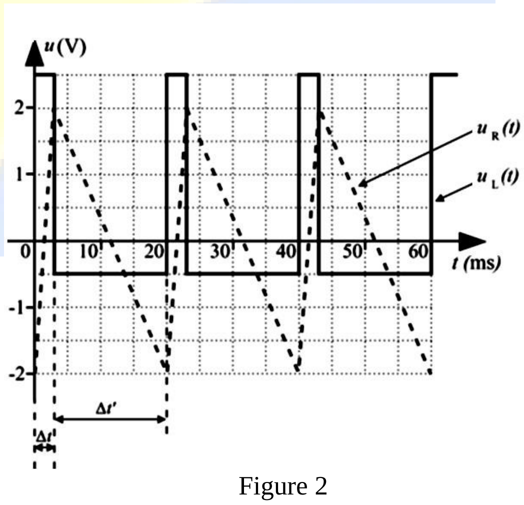


Figure 2

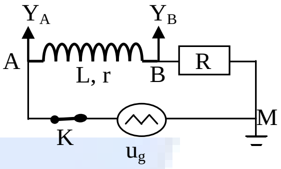


Figure 1

To determine the inductance L of the coil of negligible resistance, we carry out the circuit of figure 1. The used generator delivers, across its terminals, a triangular asymmetrical voltage u_g. The resistance of (R) is equal to 1.0 kΩ. A suitable system allows us to obtain the curves of figure 2 which represent the variation of the voltage $u_R = u_{BM}$ across (R) and that of the voltage $u_L = u_{AB}$ across the coil as a function of time.

1. How did we obtain the curve u_L by using the recorded voltages on the channels Y_A and Y_B?

2. a) Give the expression of u_L in terms of u_R.

b) i) Referring to figure 2, determine the value of the inductance L in each of the two intervals Δt and $\Delta t'$.

ii) The manufacturer announces $L \approx 2.0$ H. Comment on briefly the two values obtained of L by accepting a relative variation of absolute value 10%.

B. Effect of iron on the inductance

The set up used is that of figure 1, where we replace the generator by an ideal one of emf E = 3.2 V.

Using an appropriate device, we record the variation of the voltage $u_R = u_{BM}$ as a function of time. The origin of time is taken at the instant when the switch K is closed.

1. Derive, at an instant t, the differential equation in u_R.
2. The solution of this differential equation is of the form $u_R = U_0\,(1-e^{-t/\tau})$. Determine the expressions of the constants U_0 and τ.

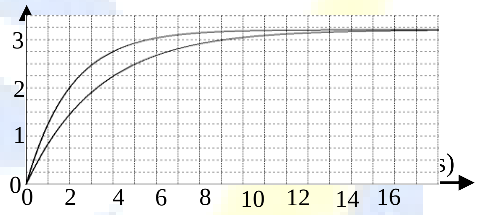


Figure 3.

3. The recording of u_R is, initially, obtained in the absence of any metal placed near the coil (curve (a)), then in the presence of a piece of iron placed near the coil (curve (b)) (fig 3).

a) Determine the values of the constants $\boldsymbol{\tau_a}$ and $\boldsymbol{\tau_b}$ associated respectively with (a) and (b).

b) i) Compare the values $\mathbf{L_a}$ and $\mathbf{L_b}$ of the inductance of the coil in the absence and in the presence of iron.

ii) What can we deduce?

C. A metal detector

The metal detector is essentially made up of an ideal (L, C) electric oscillator.

1. Draw a diagram of the circuit showing the direction of the current i at an instant t.
2. Derive the differential equation in u_C, u_C being the voltage across the capacitor.
3. Deduce, from this differential equation, the expression of the proper frequency f_0 of the oscillator in terms of L and C.
4. The detector, associated with a frequency-meter, displays, in the absence of any metal, a signal of frequency f_0= 20 kHz. The inductance of the coil being 2.0 H, calculate the value of C.
5. The previous detector displays, in the presence of a metal, a signal of frequency f = 21 kHz. Did we find iron? Justify.

Exercise III [18 pts]: Corpuscular aspect of radiations

A- Hydrogen atom

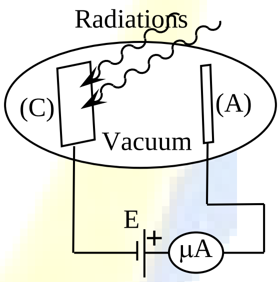
Given: $c = 2.998\times10^{8}$ m/s ; $h = 6.626\times10^{-34}$J·s; 1 eV = 1.60×10^{-19} J.

The Balmer series consists of visible spectral lines and others lying in the ultraviolet domain. The wavelength of the first line H_α is 656.2 nm, that of the second H_β is 486.1 nm, that of the third H_γ is 434.0 nm, and so on. The wavelength of the limiting spectral line of this series is 364.6 nm.

1. Calculate, in eV, the energy of a photon associated with the limiting spectral line of Balmer series.
2. Determine the energy of the starting level and that of the arrival level.

B- Photoelectric effect

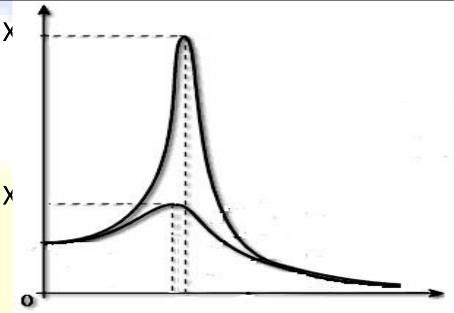
The potassium cathode (C) of a photoelectric cell has a useful surface area S = 2.00 cm^2. The cathode, whose work function is W_0 = 2.20 eV, receives radiations from a hydrogen point source placed at a distance D = 1.25 m which radiates uniformly in all directions, a power P_S = 2.00 W.

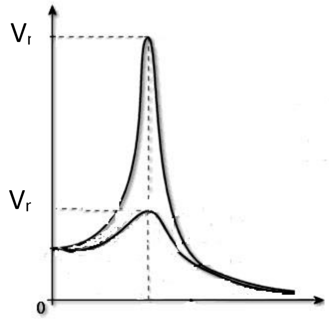


1. Calculate the threshold wavelength of the potassium cathode.
2. Which are, among the Balmer series spectral lines, the radiations able to produce a photoelectric emission?
3. The maximum kinetic energy of an emitted electron is quantized. Why?
4. Using a filter, we illuminate the cathode with the blue light H_β. The emf E of the generator is adjusted to allow the anode collecting all the electrons emitted by the cathode whose quantum efficiency is r_q = 0.875%.
   a) Show that the radiant power P_0 received by the cell is equal to 2.04×10^{-5} W.
   b) Determine the number N_0 of the incident photons on the cathode in one second;
   c) Determine the current I_0 carried by the circuit.
5. We switch off the lamp, at an instant chosen as the origin of time $t_0 = 0$. The power received by (C), at an instant t, is then written as: $P = P_0e^{-50\,t}$.
   a) Determine:
      i) The number dn of the electrons emitted by (C) between the instants t and t+dt;
      ii) The variation dq of the charge carried by the circuit between the instants t and t+dt;
   b) Deduce the expression of the current i carried by the circuit at the instant t;
   c) Determine the time at the end of which the current i will be supposed practically nil.

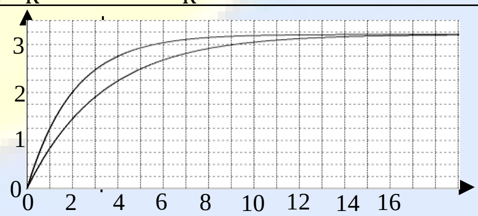
Entrance exam 2013-2014 **Solution of Physics** **14/7/2013**

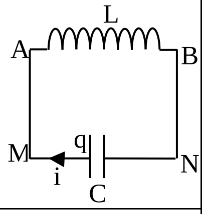
Exercise I: Forced oscillations. Phenomena of resonance.

<table>
<tr><th>Q</th><th></th><th>Notes</th></tr>
<tr><td>1.</td><td>The mechanical energy of the system (oscillator, Earth) is given by: $E_m = E_C + E_{Pé} = ½mv^2 + ½kx^2$.
$\sum P = \frac{dE_m}{dt} \Rightarrow P(\vec{R}_N) + P(\vec{f}\,) + P(\vec{F}\,) = \frac{dE_m}{dt}$
$0 - hv\cdot v + F_0\cdot\sin(\omega t+\phi)v = mv\frac{dv}{dt} + k\,x\,\frac{dx}{dt}$; while simplifying by $v = \frac{dx}{dt}$ and while replacing $\frac{dv}{dt}$ by $\frac{d^2x}{dt^2}$, we obtain: $-h\frac{dx}{dt} + F_0\cdot\sin(\omega t+\phi) = m\frac{d^2x}{dt^2} + k\,x \Rightarrow \frac{d^2x}{dt^2} + \frac{h}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{F_0}{m}\sin(\omega t+\phi)$.
Another method: $\sum\vec{F} = \frac{d\vec{P}}{dt} = m\frac{d\vec{v}}{dt} = m\frac{d^2x}{dt^2}\vec{i} = m\vec{g} + \vec{R}_N + \vec{f} + \vec{T} + \vec{F}$, with $\vec{T} = -kx\,\vec{i}$
After projection: $m\frac{d^2x}{dt^2} = -h\frac{dx}{dt} - kx + F_0\sin(\omega t+\phi) \Rightarrow \frac{d^2x}{dt^2} + \frac{h}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{F_0}{m}\sin(\omega t+\phi)$.</td><td>3</td></tr>
<tr><td>2.a)</td><td>$\frac{dx}{dt} = \omega X_m\cos(\omega t)$ et $\frac{d^2x}{dt^2} = -\omega^2 X_m\sin(\omega t)$. While replacing in the differential equation, we obtain:
$-\omega^2 X_m\sin(\omega t) + \frac{h}{m}\omega X_m\cos(\omega t) + \frac{k}{m}X_m\sin(\omega t) = \frac{F_0}{m}\sin(\omega t+\phi)$.
For $\omega t = 0 \Rightarrow \frac{h}{m}\omega X_m = \frac{F_0}{m}\sin(\phi) \Rightarrow F_0\sin(\phi) = h\omega X_m$.
For $\omega t = \pi/2 \Rightarrow -\omega^2 X_m + \frac{k}{m}X_m = \frac{F_0}{m}\sin(\pi/2+\phi) = \frac{F_0}{m}\cos(\phi) \Rightarrow -m\omega^2 X_m + k\,X_m = F_0\cos(\phi)$
$\Rightarrow [k - m\omega^2]X_m = F_0\cos(\phi) \Rightarrow \tan\phi = \frac{h\omega}{k-m\omega^2}$ et $h^2\omega^2 X_m^2 + [k - m\omega^2]^2 X_m^2 = F_0^2$
$\Rightarrow X_m = \frac{F_0}{\sqrt{h^2\omega^2+[k-m\omega^2]^2}}$</td><td>4</td></tr>
<tr><td>2.b.i</td><td>The resonance of amplitude takes place when the amplitude is maximum, i.e. when its derivative with respect to ω is nil : $dX_m/d\omega = -½F_0[2\omega h^2 - 4m\omega(k-m\omega^2)][h^2\omega^2 + (k-m\omega^2)^2]^{-3/2} = 0$
$\Rightarrow 2\omega h^2 - 4m\omega(k-m\omega^2) = 0 \Rightarrow h^2 = -2m^2\omega^2 + 2mk \Rightarrow \omega_r = \sqrt{\frac{k}{m} - \frac{1}{2}\left(\frac{h}{m}\right)^2}$</td><td>2</td></tr>
<tr><td>2.b.ii</td><td>when h increases, ω_r decreases.
</td><td>1.5</td></tr>
<tr><td>3.a</td><td>We have $x = X_m\sin(\omega t) \Rightarrow$ the expression of v : $v = \omega X_m\cos(\omega t)$.</td><td>1</td></tr>
<tr><td>3.b</td><td>The expression of the amplitude V_m of v : $V_m = \omega X_m \Rightarrow V_m = \frac{F_0}{\sqrt{h^2+[k/\omega - m\omega]^2}}$</td><td>1</td></tr>
<tr><td>3.c.i</td><td>There is a resonance of the amplitude V_m of the velocity, when the denominator is minimal, therefore for $k/\omega^2 - m = 0 \Rightarrow \omega_0 = \sqrt{\frac{k}{m}}$.</td><td>1</td></tr>
</table>

3.c	Thus, when h increases, the angular frequency of resonance ω_0 do not depend on h.  	**1.5**
4.a	The value of ϕ : $\phi = 1\times2\pi/19 = 0.33$ rad or 18.9°. The period of oscillations: $T = 19\times30 = 570$ ms or 0.570 s $\Rightarrow \omega = 2\pi/T = 11$ rad/s. $x_m = 3$ cm and $F_0 = 4\times0.3 = 1.2$ N $x = 3\sin(11\,t)$ and $F = 1.2\sin(11\,t + 0.33)$ { x en cm ; F en N and t en s}	**3**
4.b	$k - m\omega^2 = y \Rightarrow h\omega = y\tan\phi = 0.343\,y$ $h^2\omega^2 + (k-m\omega^2)^2 = h^2\omega^2 + y^2 = [F_0/X_m]^2 = 1600 \Rightarrow 1.117\,y^2 = 1600 \Rightarrow y = 37.85 = k - 0.2\times11^2$ $\Rightarrow k = 62.1$ N/m. $11\,h = 0.343\times37.85 \Rightarrow h = 1.18$ kg/s	**2**
		20

Exercise II Different roles of a coil

A.1	Since that the voltage $u_L = u_g - u_R = u_g + (-u_R)$, then it is sufficient to insert the buttons INV (of channel Y_B) and ADD**.**	**1.5**
A.2.a	According to the Ohm's law in the case of a coil, we have: $u_L = L\,di/dt$ and the Ohm's law in the case of a resistor $u_R = R\,i$, then $u_L = \frac{L}{R}\frac{du_R}{dt}$.	**2**
A.2.b.i	In the interval Δt : $\frac{du_R}{dt} = \frac{\Delta u_R}{\Delta t}$ (curve carried by a line) $\frac{du_R}{dt} = \frac{2+2}{3\times10^{-3}} = 1333$ V/s and $u_L = 2.5$ V $\Rightarrow L = \frac{2.5\times10^3}{1333} = 1.88$ H. In the interval $\Delta t'$: $\frac{du_R}{dt} = \frac{\Delta u_R}{\Delta t}$ (curve carried by a line) $\frac{du_R}{dt} = \frac{-2-2}{17\times10^{-3}} = -235.3$ V/s and $u_L = -0.5$ V $\Rightarrow L = \frac{-0.5\times10^3}{-235.3} = 2.12$ H.	**3**
A.2.b.ii	The obtained values are coherent with the value given by the manufacturer: $\frac{\Delta L_a}{L} = \frac{0.12}{2} \approx 6\% < 10\%$ And $\frac{\Delta L_b}{L} = \frac{0.12}{2} \approx 6\% < 10\%$.	**1**
B.1.	According to the law of voltage addition: $u_g = u_L + u_R \Rightarrow E = \frac{L}{R}\frac{du_R}{dt} + u_R$.	**1.5**
B.2.	$\frac{du_R}{dt} = \frac{U_0}{\tau}e^{-t/\tau}$, $E = \frac{L}{R}\frac{U_0}{\tau}e^{-t/\tau} + U_0 - U_0\,e^{-t/\tau}$. By identification and whatever the time, one obtains: $U_0 = E$ et $\frac{U_0}{\tau}\frac{L}{R} = U_0 \Rightarrow \tau = \frac{L}{R}$.	**2**
B.3.a	For $t = \tau$, $u_R = 0.63\times3.2 = 2.02$ V $\Rightarrow \tau_a = 2$ ms and $\tau_b = 3{,}3$ ms.  **Figure 3.**	**2**
B.3.b.i	$L_a = R\times\tau_a = 10^3\times2\times10^{-3} = 2$ H et $L_b = R\times\tau_b = 10^3\times3.3\times10^{-3} = 3.3$ H. $\Rightarrow$ **$L_a < L_b$.**	**1.5**
B.3.b.ii	The presence of iron near the coil causes an increase in its inductance**.**	**0.5**

C.1		**0.5**
C.2	We have $u_{AB} = u_{MN} \Rightarrow L\frac{di}{dt} = u_C$, but $i = -\frac{dq}{dt} = -C\frac{du_C}{dt} \Rightarrow -LC\frac{d^2u_C}{dt^2} = u_C$ thus : $\frac{d^2u_C}{dt^2} + \frac{1}{LC}u_C = 0$.	**2.5**
C.3	The general form of this differential equation is $\frac{d^2u_C}{dt^2} + \omega_0^2\,u_C = 0 \Rightarrow \omega_0^2 = \frac{1}{LC}$. As the proper frequency $f_0 = \omega_0/2\pi$, then $f_0 = 1/[2\pi\sqrt{LC}]$.	**1.5**
C.4	The proper frequency $f_0 = 20$ kHz $= 1/[2\pi\sqrt{LC}] \Rightarrow LC = 6.33\times10^{-11} \Rightarrow C = 3.16\times10^{-11}$ F.	**1.5**
C.5	With a frequency of 21 kHz > 20 kHz $\Rightarrow$ L' < L; this implies that L has decreased. Thus iron was not found.	**1**
		22

Exercise III Corpuscular aspect of radiations

A.1	We have $E = \frac{hc}{\lambda} = \frac{6.626\times10^{-34}\cdot 2.998\times10^{8}}{364.6\times10^{-9}}$ = 5.448×10^{-19} J and $E = \frac{5.448\times10^{-19}}{1.60\times10^{-19}}$ = 3.40 eV.	**2.5**
A.2	The electronic transition corresponding to the emission of this photon is from the level of ionization to the first excited level (n = 2) of the hydrogen atom. The starting level has by convention a zero energy and the level of arrival is such as : $E = E_\infty - E_2 \Rightarrow 3.40 = 0 - E_2 \Rightarrow E_2 = -3.40$ eV.	**2**
B.1	The threshold wavelength λ_S of the potassium cathode is such that $W_0 = hc/\lambda_S \Rightarrow \lambda_S = hc/W_0$. $\lambda_S = \frac{6.626\times10^{-34}\cdot 2.998\times10^{8}}{2.20\cdot 1.60\times10^{-19}}$ = 5.65×10^{-7} m or 565 nm.	**1.5**
B.2	The lines of the Balmer series that can cause a photoelectric emission may verify the relation $\lambda < \lambda_S$, thus only H_α, which has λ = 656.2 nm > λ_S do not cause the emission of photoelectrons. All the other lines verify $\lambda < \lambda_S$.	**1**
B.3	We have, according to the relation of Einstein, the energy of the received photon: $E = W_0 + E_{C(max)}$. As W_0 is a constant of the metal, and as E is quantized thus the maximum kinetic energy $E_{C(max)}$ is quantized.	**1.5**
B.4.a	The radiant power P_0 received by the cell: $P_0 = P_S \times s/4\pi D^2 = \frac{2\times2.0\times10^{-4}}{4\cdot\pi\cdot1.25^2}$ = 2.04×10^{-5} W.	**1.5**
B.4.b	The number N_0 of the incident photons on the cathode in one second is equal to: $N = \frac{\text{énergie des photons émis en 1 s}}{\text{énergie d'un photon}}$. $N = \frac{2.04\times10^{-5}\cdot 1}{4.09\times10^{-19}}$ = 4.99×10^{13} photons emitted in 1 s.	**1.5**
B.4.c	The number of electrons emitted during 1 s is : $N_e = r_q\cdot N$ = 0,00875·4.99×10^{13} = 4.37×10^{11} electrons emitted in 1 s. $I_0 = q/t = \frac{N_e\cdot q_e}{t} = \frac{4.37\times10^{11}\cdot1.60\times10^{-19}}{1}$ = 6.99×10^{-8} A	**2**
B.5.a.i	Knowing that $P = \frac{dW}{dt}$, dW being the energy received by (C) during dt ; and $dW = dN\cdot E = dN\cdot h\nu$. The number dn of electrons emitted, during dt, at the instant t is given by: $dn = r_q\cdot dN = r_q \frac{P}{E}\cdot dt = r_q \frac{P_0 e^{-50\,t}}{E}\cdot dt$	**1**
B.5.a.ii	The variation dq of the charge between the instants t and t + dt, is: $\lvert dq\rvert = \lvert dn\rvert\cdot e$; $\lvert dq\rvert = r_q \frac{P_0 e^{-50\,t}}{E}\cdot e\cdot dt$	**1**
B.5.b	$i = \lvert dq/dt\rvert = e\lvert dn/dt\rvert$ $i = \frac{r_q\cdot e\cdot P_0}{E} e^{-50\,t} = I_0\cdot e^{-50\,t}$.	**1**
B.5.c	i becomes practically nil when $\Delta t \approx 5\tau$, with τ = 1/50 = 0.02 s $\Rightarrow \Delta t$ = 0.10 s.	**1.5**
		18