UNIVERSITE LIBANAISE

FACULTE DE GENIE

الجامعة اللبنانية
كلية الهندسة

Entrance Exam 2011-2012 **Physics** **Duration: 2 h**

3 July 2011

I- [4 pts] Potassium - argon dating of rocks

At the time of a volcanic eruption, the lava in contact with air loses argon 40. At the instant of the eruption ($t_0 = 0$), the lava thus does not contain argon. The analysis of a non fissured block of basalt, at the instant t, shows that it contains 1.4900 mg of potassium 40 ($^{40}_{19}K$) and 0.0218 mg of argon 40 ($^{40}_{18}Ar$). Potassium 40 is radioactive; 89.3% of the mass of the disintegrated potassium 40 are transformed into calcium 40 ($^{40}_{20}Ca$) and 10.7% into argon 40 per orbital electron capture, i.e., the capture by the nucleus of an electron of the internal orbit of the electronic cloud by transforming one of the protons of the nucleus into a neutron. This capture is accompanied by the emission of photons.

1. Write down, specifying the laws used, the equations of these two transformations.
2. To what kind of radioactivity does this electronic capture resemble? Justify the answer.
3. Calculate the maximum kinetic energy KE of the emitted particle by the disintegration into calcium.
4. Calculate the energy carried by the photons emitted by the orbital electron capture.
5. Knowing that potassium 40 and argon 40 atoms have the same mass, determine the mass of the potassium 40 disintegrated between the instants 0 and t.
6. Determine the approximate date of the eruption.

The half-life of potassium 40 is $t_{½} = 1.26\times10^{9}$ years; 1 u $= 1.66\times10^{-27}$ kg $= 931.5$ MeV/c^2 ; 1 MeV $= 1.60\times10^{-13}$ J.
mass of nuclei: Ar-40 : 39.952509 u ; K-40 : 39.953576 u ; Ca-40 : 39.951619 u;
mass of an electron : $m_e = 5.486\times10^{-4}$ u

II- [8 pts] Exciter and forced oscillations

Consider a horizontal elastic oscillator formed of a spring, of stiffness k = 29.24 N/m and of negligible mass, attached to a solid (S), of mass m = 150 g and of centre of inertia G, which can slide on a horizontal rod. An exciter (E), attached to the free end P of the spring, may put this end into vibration. (Fig. 1)

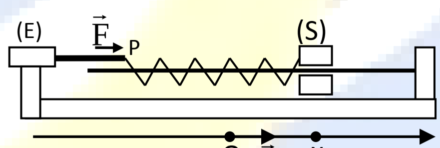


Fig.1

A) Study of the electric exciter (E)

The frequency of the exciter (E) can be suitably adjusted. The electric oscillations can be obtained using the device of the figure 2, where C = 3.0 mF and L is of adjustable value.

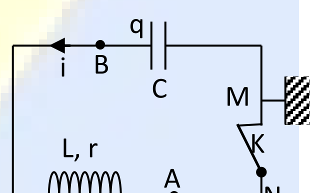


Fig.2

1. At the instant $t_0 = 0$ s, the capacitor (C) is charged with Q_0 and the switch K is closed. We can, using a convenient device, to display $u_C = u_{BM}$ and to obtain the curve of figure 3.
   a) Derive the second order differential equation in u_C.
   b) Calculate the energies stored by (C) at the instants $t_0 = 0$ s and $t_1 = 0.3$ s.

2. In order to obtain driven oscillations, we add, between A and N, a generator D.
   a) Show that the voltage u_{AN} is equal to $u_{AN} = -ri$.
   b) What value is thus given to L?
   c) Determine, from the previous calculations, the average power provided by D.

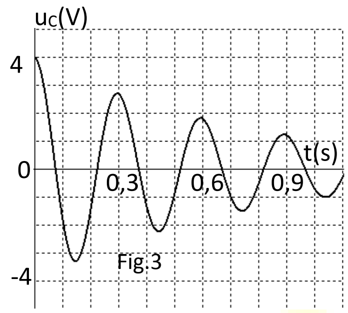


Fig.3

B) Study of the oscillator

The exciter is put on at an instant taken as an initial instant ($t_0 = 0$). At an instant t, the abscissa of G with respect to its equilibrium position is x and its velocity is $\vec{v} = v\vec{i}$.
During oscillations, (S) is supposed to be subjected to a **weak** force of friction $\vec{f} = -h\vec{v}$, with h = 0.92 kg/s, the tension in the spring $\vec{F'} = -kx\,\vec{i}$ and the oscillator is subjected to the force $\vec{F}$ exerted by the exciter: $\vec{F} = F_0 \sin(\omega' t)\,\vec{i}$ with F_0 = 1.46 N.

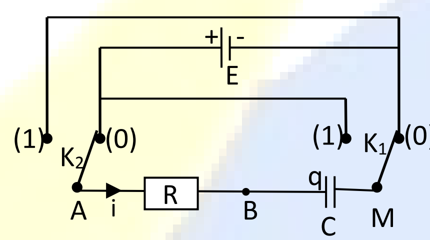
a) Show that the second order differential equation in x is: $\frac{d^2x}{dt^2} + \frac{h}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{F_0}{m}\sin(\omega' t)$

b) After a certain time, the oscillations become sinusoidal. The solution of this equation is of the form $x = A\sin(\omega' t + \varphi)$. The frequency of the exciter is adjusted so that the amplitude X_m of the oscillations is maximum. Determine,
i) an approximate value of the angular frequency ω'.
ii) the amplitude A and the phase difference φ.

III- [8 pts] Oscillations

A- Electric Oscillations

The circuit of the adjacent figure is fed by a generator of constant voltage E = 6 V; C = 1 μF and R is of adjustable value. Each of the two switches K_1 and K_2 has two possible positions 0 and 1. Consider $u_R = u_{AB}$, the voltage across the resistor, and $u_C = u_{BM}$, the voltage across the capacitor.

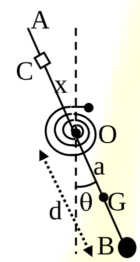


At $t_0 = 0$, the capacitor is initially discharged. Each of the two switches K_1 and K_2 is at the position (0), and $u_{AM} = +E$.
1. Derive the differential equation verified by u_C.
2. $u_C = A(1 - e^{-\frac{t}{\tau}})$ is a solution of the above differential equation. Determine the expressions of the constants A and τ.
3. Determine, in terms of R, the instant t_1 for which $u_C = +3$ V.
4. When the voltage $u_C = u_{BM}$ reaches the value $u_C = +3$ V, a device turns the two switches to the positions (1) in order to have $u_{AM} = -E$.

a) Taking $t_0 = 0$, the instant when the switches are at the positions (1), show that $u_C = -E + (E + 3)\,e^{-\frac{t}{\tau}}$.
b) At the instant t_2, the voltage u_C takes the value $u_C = -3$ V. Determine t_2 in terms of R.
5. The device turns the switches to the positions (0) when u_C takes the value $u_C = -3$ V. Calculate R, knowing that $t_1 + t_2 = 0.60$ s.

B- Mechanical oscillations

Consider a rod AB of negligible mass which can rotate about a horizontal axis (Δ) passing through O and two particles, one of mass M = 25.0 g fixed at B (OB = d = 32.6 mm) and the other of mass m = 6.0 g, placed at C, that may slide along OA (OC = x). This set constitutes a pendulum (P) of center of inertia G and of moment of inertia I with respect to (Δ). At O is fixed a spiral spring of negligible mass and stiffness $k = 2.0\times10^{-3}$ N/m. When the angular abscissa of (P), with respect to the vertical, is θ, this spring is deformed by θ and it stores an elastic potential energy EPE $= 1/2\ k\theta^2$.



(P) is shifted by a small angle and we leave it without velocity at the instant $t_0 = 0$. At an instant t, the angular abscissa of (P) is θ and its angular velocity is $\dot{\theta} = \frac{d\theta}{dt}$. ($g = 9.80$ m/s^2).

The horizontal plane through O is the gravitational potential energy reference.

1. Show that $(M + m)\,a = Md - mx$, where a = OG and that $I = mx^2 + Md^2$.

2. a) Show, by neglecting the forces of friction, that the second order differential equation in θ is written as: $\ddot{\theta} + \frac{k + (Md - mx)g}{mx^2 + Md^2}\theta = 0$ (Take: $\sin\theta = \theta$ and $\cos\theta = 1 - ½\theta^2$ for small θ in radian).

 b) i) Determine the condition that x must obey in order to obtain harmonic oscillations.
  ii) This condition being fulfilled, determine the expression of the proper period T_0 of these oscillations.
  iii) Calculate x for $T_0 = 0.60$ s.

3. In fact, the forces of friction are not negligible. The moment of these forces with respect to (Δ) is $\Gamma = -\mu\dot{\theta}$, where μ is a positive constant.

 a) Determine, at the instant t, the power due to these forces of friction.
 b) Deduce the second order differential equation in θ that describes these pseudoperiodic oscillations.
 c) The solution of this differential equation is: $\theta = 0.211\ e^{-(\mu/2I)t}\cos(10.42t - 0.10)$. Determine μ knowing that, at the end of 10 oscillations, the amplitude of the oscillations becomes 60% of its initial value.

Entrance Exam 2011-2012 **Solution of Physics** **Duration: 2 h**

3 July 2011

Exercise I- Potassium - argon dating of rocks

Q		Pts
1.	${}^{40}_{19}K \longrightarrow {}^{40}_{20}Ca + {}^{a}_{z}p$. According to the conservation laws of the charge number and mass number: A = 0 and z = -1 $\Rightarrow {}^{40}_{19}K \longrightarrow {}^{40}_{20}Ca + {}^{0}_{-1}e + {}^{0}_{0}\bar{\nu}$ ${}^{40}_{19}K + {}^{0}_{-1}e \longrightarrow {}^{40}_{18}Ar + \gamma$ + R.X.	1½
2.	**1)** It resembles the β^+ disintegration because, in the nucleus, a proton transforms to neutron.	¾
3.	**2)** Mass defect : $\Delta m = m({}^{40}_{19}K) - m({}^{40}_{20}Ca) - m({}^{0}_{-1}e)$; $\Delta m = 39.953576 - 39.951619 - 5.486\times10^{-4} = 1.408\times10^{-3}$ u ; The liberated energy: $\Delta m\times c^2 = 1.408\times10^{-3}\times931.5 = 1.31$ MeV. $KE(\beta^-) = \Delta m\times c^2 + KE(K) - KE(Ca) - E({}^{0}_{0}\bar{\nu})$; $KE(\beta^-)$ is maximum for $E({}^{0}_{0}\bar{\nu}) = 0$, KE(K) = 0 and KE(Ca) = 0 so : $KE(\beta^-)_{max} = \Delta m\times c^2 = 1.31$ MeV.	3
4.	**3)** Mass defect : $\Delta m = m({}^{40}_{19}K) + m({}^{0}_{-1}e) - m({}^{40}_{18}Ar)$; $\Delta m = 39.953576 + 5.486\times10^{-4} - 39.952509 = 1.616\times10^{-3}$ u ; The liberated energy: $\Delta m\times c^2 = 1.616\times10^{-3}\times931.5 = 1.505$ MeV.	2¼
5.	**4)** Mass of potassium 40 that transformed into argon = mass of argon = 0.0218 mg , representing 10.7% of the mass m' of potassium that disintegrated. So $m' = 0.0218\,\frac{100}{10.7} = 0.2037$ mg.	2¼
6.	The total initial mass of potassium 40 at the instant of eruption is : $m_0 = 0.2037 + 1.4900 = 1.6937$ mg. We know that $m = m_0e^{-\lambda t} \Rightarrow \ell n\frac{m}{m_0} = -\lambda t \Rightarrow -0.12814 = -\lambda t \Rightarrow -0.12814 = -0.693\frac{t}{T}$ $\Rightarrow t = 0.1849\,T = 2.33\times10^{8}$ years	2¼
		12

II- A) Study of the exciter

Q		Pts
1.a	$u_{BM} = u_{BN} \Rightarrow u_C = L\frac{di}{dt} + ri$, but $i = -\frac{dq}{dt} = -C\frac{du_C}{dt}$ et $\frac{di}{dt} = -C\frac{d^2u_C}{dt_2}$ Et $u_C = -LC\frac{d^2u_C}{dt^2} - rC\frac{du_C}{dt} \Rightarrow$ differential equation : $\frac{d^2u_C}{dt_2} + \frac{r}{L}\frac{du_C}{dt} + \frac{1}{LC}u_C = 0$	3
1.b	The energies stored in the capacitor: $W(t_0) = ½\ 3\times10^{-3}\times16 = 24\times10^{-5}$ J and $W(t_1) = ½\ 3\times10^{-3}\times(2.7)^2 = 10.94\times10^{-5}$ J.	3
2.a	According to addition of voltages : $u_{BM} = u_{BA} + u_{AN}$ $\Rightarrow u_C = -L\,C\frac{d^2u_C}{dt_2} + ri + u_{AN} = 0$ so, to obtain driven oscillations, we should have $ri + u_{AN} = 0 \Rightarrow u_{AN} = -ri$.	2¼
2.b	weak damping : $T \approx T_0 = 2\pi\sqrt{LC}$; $\left(\frac{0.3}{2\pi}\right)^2 = L\times3\times10^{-3} \Rightarrow L = 0.76$ H.	3
2.c	The average power furnished by D to compensate the power lost in the circuit: $P_{av.} = \frac{\Delta E}{\Delta t}$ $P_{av.} = (24\times10^{-3} - 10.94\times10^{-5})/0.3\times10^{-3} = 4.35\times10^{-2}$ W.	2¼
		13½

II- B) Study of the oscillator

Q		Pts
a.	According to Newton's second law : $\sum \vec{F} = \frac{d\vec{P}}{dt}$: $\frac{md\vec{v}}{dt} = -kx\,\vec{i} - h\vec{v} + \vec{F}$ $\sin(\omega' t)\,\vec{i} + m\vec{g} + \vec{R}$, by projection : $\frac{d^2x}{dt^2} + \frac{h}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{F_0}{m}\sin(\omega' t)$ Or ME = $\frac{1}{2}mv^2 + \frac{1}{2}kx^2$; P = $\frac{dME}{dt} = \vec{F}\cdot\vec{v} + \vec{f}\cdot\vec{v} \Rightarrow mv\frac{dv}{dt} + kx\frac{dx}{dt} = F_0 v\sin\omega' t - hv^2$ $\Rightarrow \frac{d^2x}{dt^2} + \frac{h}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{F_0}{m}\sin(\omega' t)$	4½
i)	maximum amplitude,, so it is the resonance amplitude: $\omega' = \omega \approx \omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{29.24}{0.15}} = 13.96$ rad/s.	2¼
ii)	$\frac{dx}{dt} = A\omega\cos(\omega t + \varphi)$ and $\frac{d^2x}{dt^2} = -A\omega^2\sin(\omega t + \varphi)$, By replacing in the differential equation: $-A\omega^2\sin(\omega t + \varphi) + \frac{h}{m}A\omega\cos(\omega t + \varphi) + \frac{k}{m}A\sin(\omega t + \varphi) = \frac{F_0}{m}\sin(\omega t)$, But $-A\omega^2\sin(\omega t + \varphi) + \frac{k}{m}A\sin(\omega t + \varphi) = 0$, because $\omega^2 = \frac{k}{m}$. So $\frac{h}{m}A\omega\cos(\omega t + \varphi) = \frac{F_0}{m}\sin(\omega t)$ This relation is valid for any t: the amplitude $A = \frac{F_0}{h\omega} = \frac{1.46}{0.92\times 13.96} = 0.113$ m = 11.3 cm and The phase difference φ is $-\frac{\pi}{2}$ rad.	3¾
		10½

Exercice III - A Electric Oscillations

Q		Pts
1.	According to the law of addition of voltages: $u_{AM} = u_{AB} + u_{BM}$; $E = Ri + u_C$; $i = \frac{dq}{dt} = C\frac{du_C}{dt}$; $E = RC\frac{du_C}{dt} + u_C \Rightarrow \frac{du_C}{dt} + \frac{1}{RC}u_C = \frac{E}{RC}$.	2¼
2.	$\frac{du_C}{dt} = \frac{1}{\tau}Ae^{-\frac{t}{\tau}}$: $\frac{1}{\tau}Ae^{-\frac{t}{\tau}} + \frac{1}{RC}A(1-e^{-\frac{t}{\tau}}) = \frac{E}{RC} \Rightarrow A = E$ and $\tau = RC$.	2¼
3.	$3 = 6(1-e^{-\frac{t_1}{RC}}) \Rightarrow -\frac{t_1}{RC} = -0.693 \Rightarrow t_1 = 6.93\times10^{-7}$ R.	1½
4.a	According to the law of addition of voltages: $u_{AM} = u_{AB} + u_{BM}$; $-E = RC\frac{du_C}{dt} + u_C$; $u_C = a + be^{\alpha t}$; $\frac{du_C}{dt} = \alpha be^{\alpha t}$; $\alpha be^{\alpha t} + \frac{a}{RC} + \frac{be^{\alpha t}}{RC} = -\frac{E}{RC}$ $\Rightarrow a = -E$; and $\alpha = -\frac{1}{RC}$; $t_0 = 0 \Rightarrow u_{C0} = 3\text{ V} = -E + b \Rightarrow b = E + 3 \Rightarrow u_C = -E + (E+3)\ e^{-\frac{t}{\tau}}$	3
4.b	$-3 = -6 + (6+3)\ e^{-\frac{t_2}{RC}} \Rightarrow \frac{1}{3} = e^{-\frac{t_2}{RC}} \Rightarrow 1.099 = \frac{t_2}{RC} \Rightarrow t_2 = 1.099\times10^{-6}$ R.	1½
5	$6.93\times10^{-7}\text{ R} + 1.099\times10^{-6}\text{ R} = 0.60\text{ s} \Rightarrow R = 3.35\times10^{5}\ \Omega$.	¾
		11¼

III- B – Mechanical oscillations

Q		Pts
1.	Barycenter formula : $(M + m)\overrightarrow{OG} = M\overrightarrow{OB} + m\overrightarrow{OC}$, by projection : $(M + m)OG = Md - mx \Rightarrow (M + m)a = Md - mx$; and $I = mx^2 + Md^2$ because m and M are particles and the rod is of negligible mass.	1½
2.a	The mechanical energy of the system [(P), spring, Earth] : $ME = KE + PE_{el} + PE_g$; $PE_g = -(M+m)OG\cos\theta$; $\Rightarrow ME = ½(mx^2 + Md^2)\dot\theta^2 + ½k\theta^2 - (M + m)\,g\,a\cos\theta$. $\frac{dME}{dt} = 0$, $½(mx^2 + Md^2)2\dot\theta\ddot\theta + ½k2\dot\theta\theta + (M + m)\,g\,a\,\dot\theta\sin\theta = 0$. with $\dot\theta \neq 0$, We obtain: $(mx^2 + Md^2)\ddot\theta + k\theta + (M + m)\,g\,a\,\theta = 0. \Rightarrow \ddot\theta + \frac{k+(Md-mx)g}{mx^2+Md^2}\theta = 0$.	3
2.b.i	But $\frac{k+(Md-mx)g}{mx^2+Md^2} > 0 \Rightarrow k+(Md-mx)g > 0 \Rightarrow mx < \frac{k}{g} + Md$ $\Rightarrow x < (2\times10^{-3}/9.8 + 25\times10^{-3}\times32.6\times10^{-3})/\ 6\times10^{-3} \Rightarrow x < 0.17$ m	1½
2.b.ii	The expression of proper period $T_0 = 2\pi\sqrt{\frac{mx^2+Md^2}{k+(Md-mx)g}}$.	¾
2.b.iii	Calculate x for $T_0 = 0.60$ s $\Rightarrow \left(\frac{0.6}{2\pi}\right)^2 = \frac{6\times10^{-3}x^2 + 25\times10^{-3}\cdot32.6^2\times10^{-6}}{2\times10^{-3} + (25\times10^{-3}\times32.6\times10^{-3} - 6\times10^{-3}x)9.8}$. $\Rightarrow 9.12\times10^{-3}\,[2+(0.815 - 6x)9.8]\times10^{-3} = 6x^2 + 0.02657\times10^{-3}$ $\Rightarrow 6x^2 + 0.536x - 0.0645 = 0 \Rightarrow x_1 = 0{,}0682$ m = 6.82 cm (accepted) and $x_2 = -0.157$ (rejected).	2¼
3.a	The power of the forces of friction : $P = \Gamma\,\dot\theta = -\mu\dot\theta^2$.	¾
3.b	The differential equation: $\frac{dME}{dt} = -\mu\dot\theta^2$; $(mx^2 + Md^2)\,\dot\theta\ddot\theta + k\dot\theta\theta + (M + m)\,g\,a\,\dot\theta\,\theta = -\mu\dot\theta^2$. $\Rightarrow (mx^2 + Md^2)\,\ddot\theta + \mu\dot\theta + [k + (Md - mx)]\,g\,\theta = 0$.	1½
3.c	$\frac{\theta_{(max)(10T)}}{\theta_{(max)(0)}} = e^{-(\mu/2I)10T} \Rightarrow 0.6 = e^{\frac{-\mu T}{I}} \Rightarrow -0.51 = -5\mu\frac{T}{I}$; T = 0.6 s and $I = 25\times10^{-3}\times(3.26)^2\times10^{-4}$ $+6\times10^{-3}\times(6.8)^2\times10^{-4} = 0.543\times10^{-4}$ kg·m². $\Rightarrow \mu = \frac{0.51\times0.543\times10^{-4}}{5\times0.6} = 9.23\times10^{-6}$ mN/s.	1½
		12¾