UNIVERSITE LIBANAISE

FACULTE DE GENIE

الجامعة اللبنانية
كلية الهندسة

Entrance Exam 2010 - 2011 **Mathematics** **Duration : 3 hours**

July 03 , 2010

The distribution of grades is over 25

I- (1.5 pt) Consider the three complex numbers $a=1+i$, $b=3+2i$ and $c=1+5i$.

Prove that $\dfrac{c}{b}=a$. Deduce that $\arctan 5-\arctan\dfrac{2}{3}=\dfrac{\pi}{4}$.

II- (1.5 pt) A bag contains three tokens A , B and C such that :
A has two red faces , B has two white faces and C has one red face and one white face .
A token is selected at random from the bag and then thrown on a table .
Knowing that the visible face is red , calculate the probability that the second face is also red .

III- (4 pts) Consider the two sequences (U_n) and (V_n) defined for $n\geq 1$ by $U_n=\left(1+\dfrac{1}{n}\right)^n$ and $V_n=\left(1+\dfrac{1}{n}\right)^{n+1}$.

1- Let g be the function defined on $[0\,;+\infty[$ by $g(x)=x-\ell n(1+x)$.

a) Set up the table of variations of g . Deduce that , for all $x>0$, $\ell n(1+x)<x$.

b) Calculate $\ell n(U_n)$ and prove that , for all $n\geq 1$, $U_n<e$.

2- Let h be the function defined on $[0\,;+\infty[$ by $h(x)=\dfrac{x}{x+1}-\ell n(1+x)$.

a) Set up the table of variations of h . Deduce that , for all $x>0$, $\ell n(1+x)>\dfrac{x}{1+x}$.

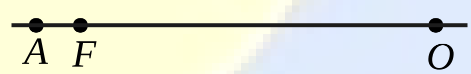
b) Calculate $\ell n(V_n)$ and prove that , for all $n\geq 1$, $V_n>e$.

3- a) Prove that , for all $n\geq 1$, $V_n-U_n=\dfrac{1}{n}U_n$. Deduce that , for all $n\geq 1$, $V_n-U_n<\dfrac{e}{n}$.

b) Prove that , for all $n\geq 1$, $0<e-U_n<V_n-U_n$.

c) Deduce that $\displaystyle\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^n=e$.

IV- (5 pts) Given 3 collinear points A , F and O such that



$AF=1$ and $FO=8$. Let (ω) be a variable circle tangent to (OA) at A .
The tangents to (ω) , other than (OA) , drawn through O and F intersect at L .

1- Prove that , as (ω) varies , L moves on an ellipse (E) to be determined .

2- The plane is referred to a direct orthonormal system $(O\,;\vec{i}\,,\vec{j}\,)$ such that $\overrightarrow{OF}=-8\,\vec{i}$.

a) Prove that $\dfrac{(x+4)^2}{25}+\dfrac{y^2}{9}=1$ is an equation of (E) .

b) Knowing that O is a focus of (E), determine an equation of the associated directrix (d).

3- a) Determine the points of intersection P and Q of (E) and the axis of ordinates . Draw (E) .

b) P being the point with positive ordinate , the tangent (Δ) to (E) at P cuts the non focal axis of (E) at T . Prove that T belongs to the auxiliary circle of (E).

4- Let $S(x_0\ ;\ y_0)$ be a point of (E) such that $y_0 \neq 0$.

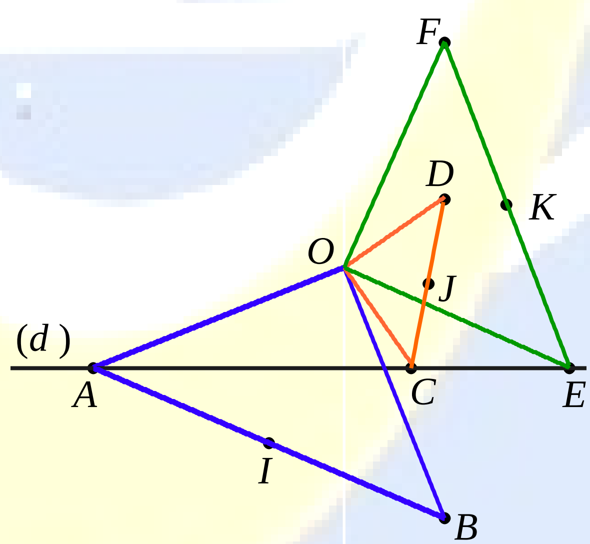
a) Prove that the tangent (δ) to (E) at S cuts the directrix (d) at the point L of ordinate $-\dfrac{9\,x_0}{4\,y_0}$.

b) The straight line (OS) cuts (E) again at a point S'. Prove that the tangent (δ') to (E) at S' cuts the directrix (d) at the same point L .

V- (6 pts) Consider in an oriented plane a straight line (d) and a point O not belonging to (d).

Consider on (d), 3 points A , C and E such that $\overrightarrow{AC}=2\overrightarrow{CE}$ and construct the right isosceles triangles OAB , OCD and OEF such that $(\overrightarrow{OA}\ ;\overrightarrow{OB})=(\overrightarrow{OC}\ ;\overrightarrow{OD})=(\overrightarrow{OE}\ ;\overrightarrow{OF})=\dfrac{\pi}{2}\quad(2\pi)$.

Let I, J and K be the mid points of $[AB]$, $[CD]$ and $[EF]$ respectively .



1- Prove that the three points B, D and F are collinear on a straight line (d_1) perpendicular to (d).

2- Determine the ratio and an angle of the similitude S of center O such that $S(A)=I$.

3- Determine $S(C)$ and $S(E)$.

4- Prove that I , J and K are collinear on a straight line (d_2) and calculate $\dfrac{IJ}{JK}$.

5- Prove that the centers of gravity of the triangles OAB, OCD and OEF are also collinear on a straight line (d_3) parallel to (d_2) .

6- Refer the plane to the direct orthonormal system $(O\ ;\vec{u}\ ,\ \vec{v})$ such that $A(-5\,;-2)$ and $C(1\,;-2)$.

a) Determine the coordinates of E .

b) Determine the complex expression of S .
c) Determine the coordinates of each of the points I , J and K and verify that these points are collinear .
d) Determine the coordinates of each of the points B , D and F and verify that these points are collinear .

VI- (7 pts) Consider the function f defined on the interval $K = [-\frac{\pi}{2} ; \frac{\pi}{2}]$ by $f(x) = e^x \cos x$.

Let (C) be the representative curve of f in an orthonormal system $(O ; \vec{i} , \vec{j})$.

1- a) Prove that $f'(x) = \sqrt{2}\, e^x \cos\left(x + \frac{\pi}{4}\right)$ then solve the equation $f'(x) = 0$ in the interval K .

b) Prove that f is strictly increasing on $[-\frac{\pi}{2} ; \frac{\pi}{4}]$ and strictly decreasing on $[\frac{\pi}{4} ; \frac{\pi}{2}]$.

c) Set up the table of variations of f . Verify that $f(\frac{\pi}{4}) \approx 1.55$.

2- a) Verify that $f'(x) = \sqrt{2}\, e^{-\frac{\pi}{4}} f(x + \frac{\pi}{4})$ and prove that $f''(x) = -2e^x \sin x$.

b) Study the concavity of (C) and determine its point of inflection I .

c) Determine an equation of the tangent (T) to (C) at I .

d) Draw (T) and (C) (***graph unit : 2 cm***) .

3- a) Determine the real numbers a and b so that the function $F : x \to (a\cos x + b\sin x)e^x$ is an antiderivative of f .

b) Calculate the area of the domain bounded by (C) and the axis of abscissas .

4- a) Prove that f has on the interval $[-\frac{\pi}{2} ; \frac{\pi}{4}]$ an inverse function g .

b) Determine the domain of definition of g .

5- Let (C') be the representative curve of g in the same system as (C) .

Prove that the tangent to (C') at the point of intersection with the axis of abscissas is parallel to (T) . Draw (C') .

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Entrance Exam 2010 - 2011 **Solution mathematics** **Duration : 3 hours**
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I- We have $a=1+i$, $b=3+2i$ and $c=1+5i$.

$$\frac{c}{b}=\frac{1+5i}{3+2i}=\frac{(1+5i)(3-2i)}{(3+2i)(3-2i)}=\frac{13+13i}{13}=1+i=a\ .$$

OR $ab=(1+i)(3+2i)=3+2i+3i-2=1+5i=c$. Therefore , $\frac{c}{b}=a$.

$\frac{c}{b}=a$ gives $\arg(\frac{c}{b})=\arg(a)$; that is $\arg(c)-\arg(b)=\arg(a)$ where

- $\arg(a)=\frac{\pi}{4}\ (2\pi)$;

- An argument of b is β such that $\beta\in]0\,;\frac{\pi}{2}[$ and $\tan\beta=\frac{\text{Im}(b)}{\text{Re}(b)}=\frac{2}{3}$ then $\arg(b)=\arctan\frac{2}{3}\ (2\pi)$
- An argument of c is γ such that $\gamma\in]0\,;\frac{\pi}{2}[$ and $\tan\gamma=\frac{\text{Im}(c)}{\text{Re}(c)}=5$ then $\arg(c)=\arctan 5\ (2\pi)$

Finally $\arg(c)-\arg(b)=\arg(a)$ gives $\arctan 5-\arctan\frac{2}{3}=\frac{\pi}{4}+2k\pi$ where $k\in Z$.

But $0<\arctan 5-\arctan\frac{2}{3}<\frac{\pi}{2}$, then $\arctan 5-\arctan\frac{2}{3}=\frac{\pi}{4}$.

II- Consider the event R : " the visible face is red " .

Since the token A is the only token that has two red faces , the required probability is $p(A/R)$.

All tokens have the same probability of being selected ; that is $p(A)=p(B)=p(C)=\frac{1}{3}$.

- If token A is selected and thrown then , the visible face is necessarily red ; that is $p(R/A)=1$.
- If token B is selected and thrown then , the visible face is necessarily white ; that is $p(R/B)=0$.
- If token C is selected and thrown then , the visible face is either white or red and $p(R/C)=\frac{1}{2}$

$p(A/R)=\frac{p(A\cap R)}{p(R)}$ where :

- $p(A\cap R)=p(A)\times p(R/A)=\frac{1}{3}\times 1=\frac{1}{3}$

- $p(R) = p(A \cap R) + p(B \cap R) + p(C \cap R)$

$p(R) = \frac{1}{3} + p(B) \times p(R \cap B) + p(C) \times p(R/C) = \frac{1}{3} + \frac{1}{3} \times 0 + \frac{1}{3} \times \frac{1}{2} = \frac{1}{2}$.

Finally , $p(A/R) = \frac{p(A \cap R)}{p(R)} = \frac{1}{3} \div \frac{1}{2} = \frac{2}{3}$.

III- 1- The function g is defined on $[0\,;+\infty[$ by $g(x) = x - \ell n(1+x)$

a) $\lim\limits_{x \to +\infty} g(x) = \lim\limits_{x \to +\infty} x\left[1 - \frac{\ell n(1+x)}{x}\right] = +\infty$.

$$g'(x) = 1 - \frac{1}{x+1} = \frac{x}{1+x} \; .$$

x	0	$+\infty$
$g'(x)$	0 +	
$g(x)$	0 ↗	$+\infty$

The function g admits 0 as an absolute minimum when $x = 0$;
therefore , for all $x > 0$, $g(x) > 0$; that is $\ell n(1+x) < x$.

b) ▪ $\ell n(U_n) = n\,\ell n\left(1 + \frac{1}{n}\right)$.

▪ For $x = \frac{1}{n}$, $\ell n(1+x) < x$ gives $\ell n\left(1 + \frac{1}{n}\right) < \frac{1}{n}$; therefore $\ell n(U_n) < 1$. Hence $U_n < e$.

2- The function h is defined on $[0\,;+\infty[$ by $h(x) = \frac{x}{x+1} - \ell n(1+x)$.

a) $\lim\limits_{x \to +\infty} \frac{x}{x+1} = 1$; then $\lim\limits_{x \to +\infty} h(x) = -\infty$.

$$h'(x) = \frac{1}{(x+1)^2} - \frac{1}{x+1} = \frac{-x}{(x+1)^2} \; .$$

x	0	$+\infty$
$h'(x)$	0 −	
$h(x)$	0 ↘	$-\infty$

The function h admits 0 as an absolute maximum when $x = 0$;
therefore , for all $x > 0$, $h(x) < 0$; that is $\ell n(1+x) > \frac{x}{x+1}$.

b) ▪ $\ell n(V_n) = (n+1)\,\ell n\left(1 + \frac{1}{n}\right)$.

▪ For $x = \frac{1}{n}$, $\ell n(1+x) > \frac{x}{x+1}$ gives $\ell n\left(1 + \frac{1}{n}\right) > \frac{1}{n+1}$; therefore $\ell n(V_n) > 1$. Hence $V_n > e$.

3- a) ▪ For all $n \geq 1$, $V_n - U_n = \left(1+\frac{1}{n}\right)^{n+1} - \left(1+\frac{1}{n}\right)^{n} = \left(1+\frac{1}{n}\right)^{n}\left(1+\frac{1}{n}-1\right) = \frac{1}{n}U_n$.

▪ The relation $U_n < e$ gives $\frac{1}{n}U_n < \frac{e}{n}$ then , $V_n - U_n < \frac{e}{n}$.

b) ▪ The relation $U_n < e$ gives $e - U_n > 0$.

▪ The relation $V_n > e$ gives $V_n - U_n > e - U_n$.

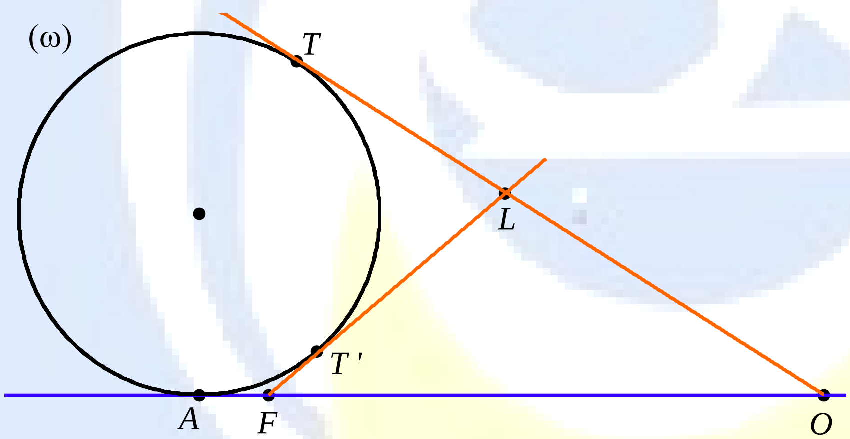
Therefore , for all $n \geq 1$, $0 < e - U_n < V_n - U_n$.

c) ▪ The relation $0 < e - U_n < V_n - U_n$ where $V_n - U_n < \frac{e}{n}$ gives $0 < e - U_n < \frac{e}{n}$

▪ $\lim\limits_{n \to +\infty} \frac{e}{n} = 0$ then , $\lim\limits_{n \to +\infty} (e - U_n) = 0$; that is $\lim\limits_{n \to +\infty} U_n = e$; $\lim\limits_{n \to +\infty} \left(1+\frac{1}{n}\right)^{n} = e$

IV- 1- We have $OT = OA = 9$, $FT' = FA = 1$ and $LT = LT'$.

$LO = OT - LT = 9 - LT$ and $LF = LT' + FT' = LT' + 1$. Therefore $LO + LF = 10 > FO$.



As (ω) varies , L moves on the ellipse (E) of foci O and F and of major axis length $2a = 10$.

2- a) In the given system $O(0\,;0)$ and $F(-8\,;0)$.

▪ The center of the ellipse is the mid point $(-4\,;0)$ of $[OF]$.

▪ The focal axis is the axis of abscissas .

▪ $2a = 10$ and $2c = OF = 8$; then $b = \sqrt{a^2 - c^2} = 3$.

An equation of this ellipse (E) is $\frac{(x+4)^2}{25} + \frac{y^2}{9} = 1$.

b) For the ellipse (E), the focal axis is the axis of abscissas .

The directrix corresponding to the focus O is the straight line (d) of equation $x = -4 + \frac{a^2}{c} = \frac{9}{4}$.

3- a) ▪ The axis of ordinates is the perpendicular to the focal axis at the focus O of (E) ;

therefore it cuts (E) at two points P and Q such that $\overline{OP} = -\overline{OQ} = p = \frac{9}{5}$.

OR ▪ The ordinates of the points of intersection of the axis of ordinates and (E) are the solutions of the

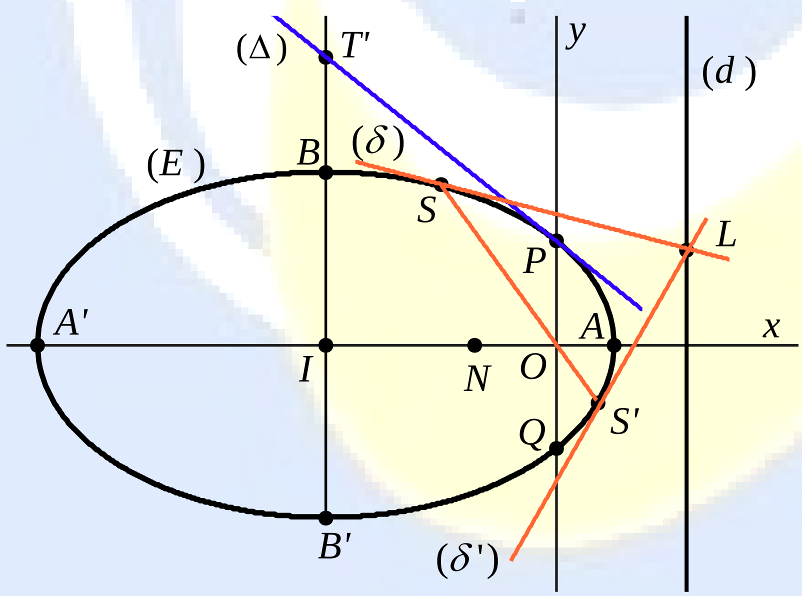
the equation $9(0+4)^2 + 25y^2 = 225$; $25y^2 = 81$; $y = \frac{9}{5}$ or $y = -\frac{9}{5}$.

Hence $P(0; \frac{9}{5})$, $Q(0; -\frac{9}{5})$.

▪ The principal vertices of (E) are the points of the focal axis of abscissas $-4 + a = 1$ and $-4 - a = -9$; these points are $A(1\,;\,0)$ and $A'(-9\,;\,0)$.

▪ The secondary vertices of (E) are the points with abscissa -4 of ordinates $b = 3$ and $-b = -3$; these points are $B(-4\,;\,3)$ and $B'(-4\,;\,-3)$.

▪ Drawing (E).



b) An equation of the tangent (Δ) to (E) at $P(0;\frac{9}{5})$ is $\frac{(x_P+4)}{25}(x+4)+\frac{y_P}{9}y=1$

(Δ) : $4x+5y-9=0$.
(Δ) cuts the non focal axis of (E) at $T(-4\,;5)$
The auxiliary circle of (E) is the circle (γ) of center $I(-4\,;0)$ and radius $a=5$.
$IT=5$ then $T\in(\gamma)$.

4- a) An equation of the tangent (δ) to (E) at $S(x_0\,;y_0)$ is $\frac{(x_0+4)}{25}(x+4)+\frac{y_0}{9}y=1$.

(δ) cuts the directrix (d) at the point $L(\frac{9}{4}\,;y)$ such that $\frac{(x_0+4)}{25}(\frac{9}{4}+4)+\frac{y_0}{9}y=1$; then $y=-\frac{9x_0}{4y_0}$.

b) The straight line (OS) cuts (E) again at a point $S'(x_1\,;y_1)$ such that $\frac{x_1}{y_1}=\frac{x_0}{y_0}$

since O , S and S' are collinear .

The tangent (δ') to (E) at S' cuts the directrix (d) at the point of ordinate $-\frac{9x_1}{4y_1}=-\frac{9x_0}{4y_0}$

which is the point L .

V- 1- The triangle OAB is direct and right isosceles at O ; then $OA=OB$ and $(\overrightarrow{OA}\,;\overrightarrow{OB})=\frac{\pi}{2}\quad(2\pi)$.

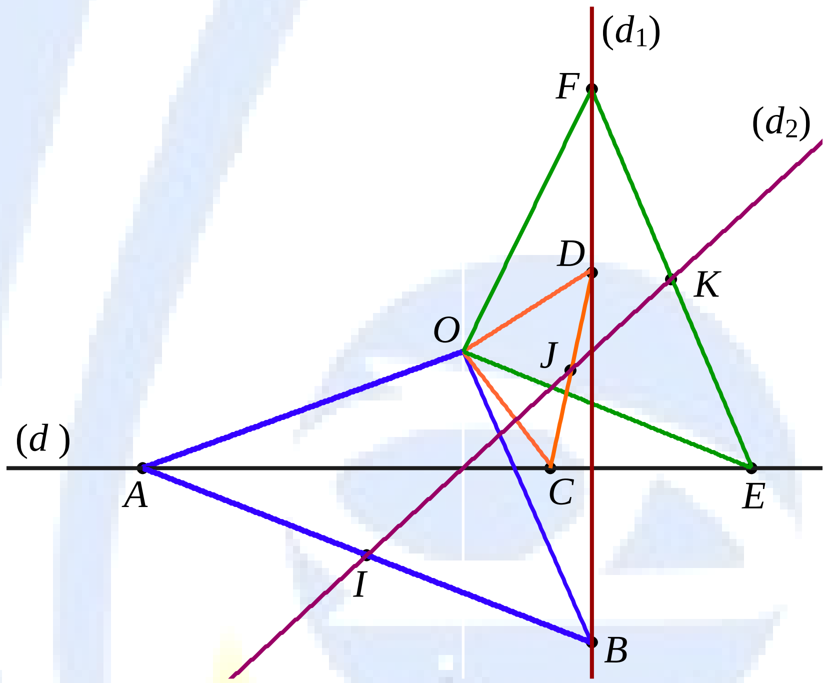
Therefore $B=r(A)$ where r is the rotation of center O and angle $\frac{\pi}{2}$ radians .

Similarly , $D=r(C)$ and $F=r(E)$.
The points A , C and E belong to the straight line (d); therefore their images B , D and F by r belong to the image of (d) by r which is a straight line (d_1) perpendicular to (d) since the angle of r is $\frac{\pi}{2}$

2- The triangle OAB is direct and right isosceles at O and I is the mid point of $[AB]$; then

$OI=\frac{\sqrt{2}}{2}OA$ and $(\overrightarrow{OA}\,;\overrightarrow{OI})=\frac{\pi}{4}\quad(2\pi)$. Therefore , the similitude S of center O that transforms A into I has the ratio $\frac{\sqrt{2}}{2}$ and an angle of $\frac{\pi}{4}$ radians .

3- The triangle OCD is also direct and right isosceles at O and J is the mid point of $[CD]$. Therefore $(\overrightarrow{OC}\,;\overrightarrow{OJ}) = \frac{\pi}{4}$ and $OJ = \frac{\sqrt{2}}{2}OC$. Hence $S(C) = J$.
Similarly , $S(E) = K$.



4- ▪ The points A , C and E belong to the straight line (d) ; then their images I , J and K by S , belong to the image of (d) by S which a straight line (d_2) .

▪ $S(A) = I$, $S(C) = J$ and $S(E) = K$ then $IJ = \frac{\sqrt{2}}{2}AC$ and $JK = \frac{\sqrt{2}}{2}CE$. Therefore $\frac{IJ}{JK} = \frac{AC}{CE} = 2$.

OR The points A , C and E are such that $\overrightarrow{AC} = 2\overrightarrow{CE}$; then their images I , J and K by S , are such that $\overrightarrow{IJ} = 2\overrightarrow{JK}$; therefore I , J and K are collinear and $\frac{IJ}{JK} = 2$.

5- Let P , Q and R be the respective centers of gravity of the triangles OAB , OCD and OEF .
These points are such that $\overrightarrow{OP} = \frac{2}{3}\overrightarrow{OI}$, $\overrightarrow{OQ} = \frac{2}{3}\overrightarrow{OJ}$ and $\overrightarrow{OR} = \frac{2}{3}\overrightarrow{OK}$.

Hence P , Q and R are the respective images of I , J and K by the dilation h of center O and ratio $\frac{2}{3}$

The points I , J and K belong to the straight line (d_2) ; then their images P , Q and R by h belong to the image of (d_2) by h which a straight line (d_3) parallel to (d_2) .

6- Refer the plane to the direct orthonormal system $(O\ ;\vec{u}\ ,\ \vec{v})$ such that $A(-5\ ;-2)$ and $C(1\ ;-2)$.

a) Let $E(x\ ;\ y)$. The relation $\overrightarrow{AC} = 2\overrightarrow{CE}$ is equivalent to $2(x-1)=6$ and $2(y+2)=0$; then $x=4$ and $y=-2$. Finally , $E(4\ ;-2)$.

b) $S = S\left(O\ ;\ \frac{\sqrt{2}}{2}\ ;\ \frac{\pi}{4}\right)$; the complex expression of S is $z' = \frac{\sqrt{2}}{2}e^{i\frac{\pi}{4}}z = \frac{1}{2}(1+i)z$.

c) $I = S(A)$; then $z_I = \frac{1}{2}(1+i)z_A = \frac{1}{2}(1+i)(-5-2i) = -\frac{3}{2}-\frac{7}{2}i$. Therefore $I\left(-\frac{3}{2}\ ;-\frac{7}{2}\right)$.

$J = r(C)$; then $z_J = \frac{1}{2}(1+i)z_C = \frac{1}{2}(1+i)(1-2i) = 2+i$. Therefore $J\left(\frac{3}{2}\ ;-\frac{1}{2}\right)$.

$K = r(E)$; then $z_K = \frac{1}{2}(1+i)z_E = \frac{1}{2}(1+i)(4-2i) = 2+4i$. Therefore $K(3\ ;1)$.

Therefore I , J and K are collinear on the straight line of equation $y = x-2$.

OR $\overrightarrow{IJ}(3\ ;3)$ and $\overrightarrow{JK}(1.5\ ;1.5)$; then $\overrightarrow{IJ} = 2\overrightarrow{JK}$; therefore I , J and K are collinear .

d) $I\left(-\frac{3}{2}\ ;-\frac{7}{2}\right)$ is the mid point of $[AB]$ where $A(-5\ ;-2)$; therefore $B(2\ ;-5)$.

$J\left(\frac{3}{2}\ ;-\frac{1}{2}\right)$ is the mid point of $[CD]$ where $C(1\ ;-2)$; therefore $D(2\ ;1)$.

$K(3\ ;1)$ is the mid point of $[EF]$ where $E(4\ ;-2)$; therefore $F(2\ ;4)$

OR $r = r\left(O\ ;\ \frac{\pi}{2}\right)$; the complex expression of r is $z' = iz$.

$B = r(A)$; then $z_B = iz_A = i(-5-2i) = 2-5i$; therefore $B(2\ ;-5)$.

Similarly , $D = r(C)$ and $F = r(E)$; therefore $D(2\ ;1)$ and $F(2\ ;4)$.

The three points B , D and F are collinear on the straight line of equation $x = 2$.

VI- $f(x)=e^{x}\cos x$, $x\in[-\frac{\pi}{2};\frac{\pi}{2}]$.

1- a) $f'(x)=e^{x}\cos x-e^{x}\sin x=e^{x}(\cos x-\sin x)$ and

$$\sqrt{2}\,e^{x}\cos\left(x+\frac{\pi}{4}\right)=\sqrt{2}\,e^{x}\,(\cos x\cos\frac{\pi}{4}-\sin x\sin\frac{\pi}{4})=\sqrt{2}\,e^{x}\,(\frac{\sqrt{2}}{2}\cos x-\frac{\sqrt{2}}{2}\sin x\,)=e^{x}\,(\cos x-\sin x\,).$$

Therefore $f'(x)=\sqrt{2}\,e^{x}\cos\left(x+\frac{\pi}{4}\right)$.

The equation $f'(x)=0$ is equivalent to $\cos\left(x+\frac{\pi}{4}\right)=0$ where $-\frac{\pi}{4}\le x+\frac{\pi}{4}\le\frac{3\pi}{4}$; therefore

$x+\frac{\pi}{4}=\frac{\pi}{2}$; $x=\frac{\pi}{4}$.

b) If $x\in[-\frac{\pi}{2};\frac{\pi}{4}]$ then , $x+\frac{\pi}{4}\in[-\frac{\pi}{4};\frac{\pi}{2}]$; therefore $\cos\left(x+\frac{\pi}{4}\right)\ge 0$ and f is strictly increasing ;

If $x\in[\frac{\pi}{4};\frac{\pi}{2}]$ then , $x+\frac{\pi}{4}\in[\frac{\pi}{2};\frac{3\pi}{4}]$; therefore $\cos\left(x+\frac{\pi}{4}\right)\le 0$ and f is strictly decreasing .

c) Table of variations of f :

$m=f(\frac{\pi}{4})=\frac{\sqrt{2}}{2}e^{\frac{\pi}{4}}\approx 1.55$

x	$-\frac{\pi}{2}$		$\frac{\pi}{4}$		$\frac{\pi}{2}$
$f'(x)$		+	0	−	
$f(x)$	0	↗	m	↘	0

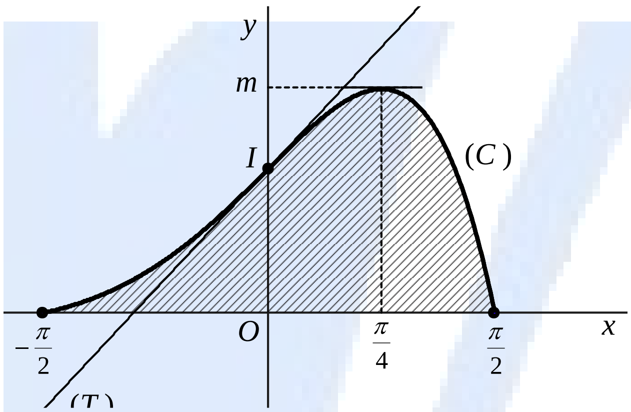
2- a) ▪ $f'(x)=\sqrt{2}\,e^{x}\cos\left(x+\frac{\pi}{4}\right)=\sqrt{2}\,e^{-\frac{\pi}{4}}\times e^{x+\frac{\pi}{4}}\cos(x+\frac{\pi}{4})=\sqrt{2}\,e^{-\frac{\pi}{4}}f(x+\frac{\pi}{4})$.

▪ $f''(x)=\sqrt{2}\,f'(x+\frac{\pi}{4})=2e^{x}\cos\left(x+\frac{\pi}{2}\right)=-2e^{x}\sin x$.

b) The sign of $f''(x)$ is the opposite to that of $\sin x$; therefore :

▪ If $x\in[-\frac{\pi}{2};0[$ then $f''(x)>0$ and (C) concaves upwards .

▪ If $x\in]0;\frac{\pi}{2}]$ then $f''(x)<0$ and (C) concaves downwards .



The point of inflection of (C) is $I(0\,;1)$.

c) An equation of the tangent (T) to (C) at I is $y = f'(0)x+1$; (T) : $y = x+1$.

d) Drawing (T) and (C) .

3- a) $F(x) = (a\cos x + b\sin x)e^x$; $F'(x) = (a\cos x + b\sin x)e^x + (-a\sin x + b\cos x)e^x$.

The function F is an antiderivative of f if , for all x in $[-\frac{\pi}{2}\,;\frac{\pi}{2}]$, $F'(x) = f(x)$; that is

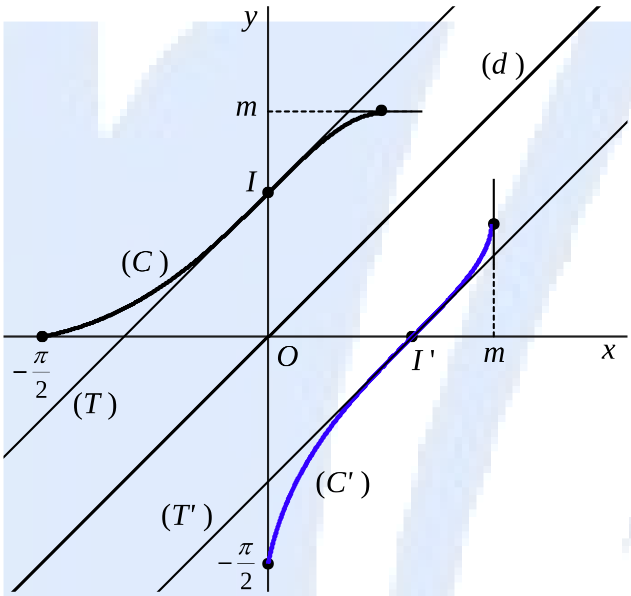
$(a+b)\cos x - (a-b)\sin x = \cos x$; therefore , $a+b=1$ and $a-b=0$. Finally , $a=b=\frac{1}{2}$.

b) The curve (C) lies above the axis of abscissas , the required area is $S = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)\,dx$ *units of area* .

$S = F(\frac{\pi}{2}) - F(-\frac{\pi}{2}) = \frac{1}{2}e^{\frac{\pi}{2}} + \frac{1}{2}e^{-\frac{\pi}{2}}$ *units of area* ; $S = 2(e^{\frac{\pi}{2}} + e^{-\frac{\pi}{2}})$ cm^2 .

4- a) The function f is continuous on $[-\frac{\pi}{2}\,;\frac{\pi}{4}]$ (product of two continuous functions) and strictly increasing ; therefore f has an inverse function g whose domain of definition is

$f\left([-\frac{\pi}{2}\,;\frac{\pi}{4}]\right) = [\,0\,;\,m\,]$.



b) (C) cuts the axis of ordinates at $I(0\,;1)$ and has at this point a tangent line (T) parallel to (d). By symmetry with respect to (d), (C') cuts the axis of abscissas at $I'(0\,;1)$ and has at this point a tangent line (T') parallel to (d) ; is (T') is parallel to (T) .

5) The representative curve (C') of g is the symmetric of the part of (C) in $[-\frac{\pi}{2}\,;\frac{\pi}{4}]$ with respect to the straight line (d) of equation $y = x$.