UNIVERSITE LIBANAISE

FACULTE DE GENIE

الجامعة اللبنانية
كلية الهندسة

Entrance Exam 2010-2011 **Physics** **Duration: 2 hours**
04 July 2010

I- [6 pts] Torsion pendulum

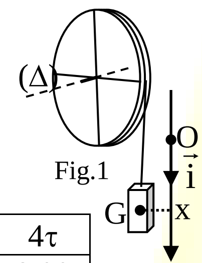


Fig.1

A cylindrical wheel, of radius R = 10 cm, can rotate about its horizontal axis (Δ), I_0 being its moment of inertia with respect to (Δ). A block (B), of mass m = 50 g and of center of inertia G, is hung from a string fixed to the wheel and wound on its periphery. Some positions occupied by G, at equal time intervals τ = 50 ms, are located by the abscissa x of G along a vertical axis (O, $\vec{i}$). The adjacent table gives, at specified instants, the value of x and the value $v = \frac{dx}{dt}$ of the velocity $\vec{v}$ of G. Take $g = 10\ m/s^2$.

t	0	τ	2τ	3τ	4τ
x (cm)	0	0.50	2.00	4.50	8.00
v (m/s)	0	0.20		0.60	0.80

1. a) . Calculate, with justification, the value of v at the instant 2τ.
   b) Draw the variation of v as a function of time.
   c) Deduce that, at the instant t, the value of the linear momentum P of (B) is of the form P = mat, a being a constant to be determined.
   d) Show that the expression of the tension $\vec{F}$ in the string is given by F = m (g - a). Calculate its value.

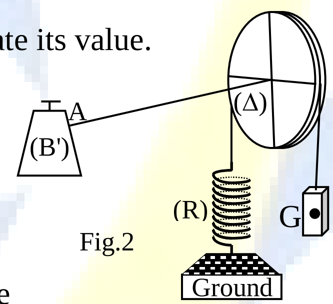


Fig.2

2. Knowing that x = Rθ, θ being the angular abscissa of the wheel at the same instant t, show, by applying the theorem of angular momentum, that $I_0 = 7.5\times10^{-4}\ kg\cdot m^2$.
3. The wheel is made to stop; it is brought into equilibrium using a vertical spring (R), of stiffness k = 5.0 N/m, the upper end of (R) being attached to a string fixed to the periphery of the wheel.
   a) Determine, at equilibrium, the elongation $\Delta\ell$ of the spring.
   b) At the center of the wheel is fixed a horizontal torsion wire, of torsion constant C, whose other end is fixed to a block stop (B'). At equilibrium, the torsion wire is not twisted. The device thus obtained is put into oscillation around its equilibrium position. Knowing that the forces of friction are negligible:
      i) show that the differential equation in θ can be written as: $\ddot{\theta} + \frac{kR^2 + C}{I_0 + mR^2}\theta = 0$
      ii) deduce the expression of the natural (proper) period T_0 of oscillations.
   c) Figure 3 shows the variation of θ as a function of time t. Determine then the value of C.

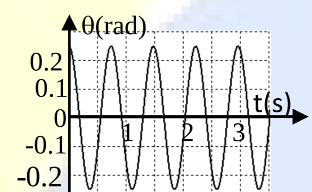


Fig.3

II- [7 pts] Analogy

A- Determination of the characteristics L and r of a coil

In order to determine the characteristics L and r of a coil, we set up the circuit of figure 4, where

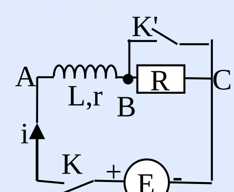
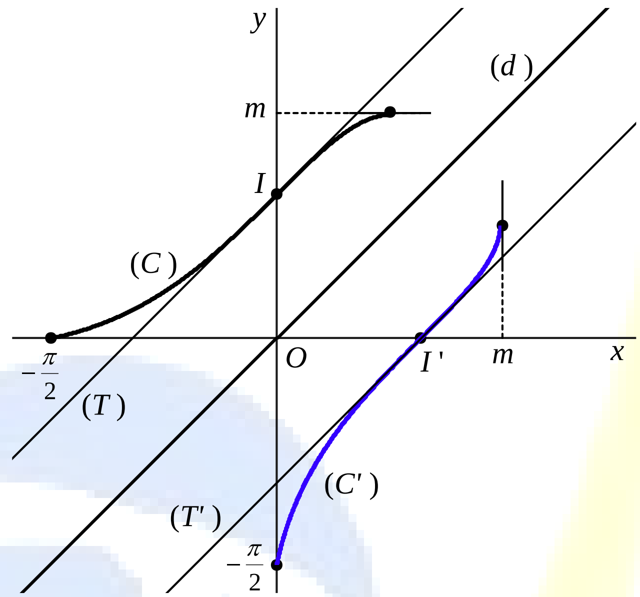


Fig. 4



$R = 1\ k\Omega$ and E the emf of the generator. At the instant $t_0 = 0$, we close K (K' remains open). At an instant t, the circuit carries a current i.

1. a) Derive the differential equation verified by the current i.

b) The solution of this equation is given by $i = a + b\,e^{-\frac{t}{\tau}}$. Determine, in terms of the data, the expressions of the constants a, b and τ.

c) Deduce the expression of the current I_0 in the steady-state.

2. Using a suitable device (D), we obtain the graph of figure 5. Determine graphically, with justification, the value of I_0 and that of the time constant τ of the RL series circuit.

3. We repeat the experiment with K' closed. We obtain the graph of figure 6. Determine, from the two graphs, the value of E, that of r and that of L.

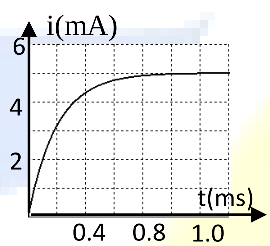


Fig.5

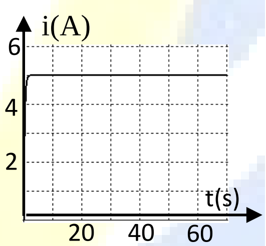


Fig.6

B-Phenomenon of electromagnetic induction

The device of Laplace's rails, of negligible resistance, lies in a horizontal plane. A rod CD, of resistance R, perpendicular to the rails, has a length $\ell = 10$ cm and a mass m = 10 g (figure 7).

This device is placed in a uniform and vertical magnetic field $\vec{B}$ of magnitude B = 0.3 T

At the instant $t_0 = 0$, the rod being in C_0D_0, we apply at its center of inertia G a constant force $\vec{F} = F\,\vec{j}$. At an instant t, the abscissa of G is y = OG, its velocity is $\vec{v} = v\,\vec{j}$ and its linear momentum is $\vec{P} = m\vec{v} = P\,\vec{j}$.

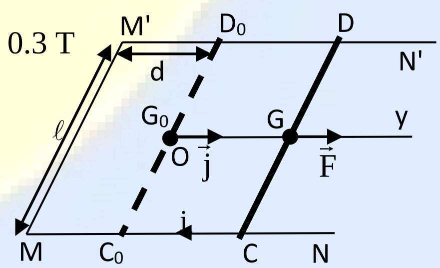


Fig.7

1. Knowing that the real direction of the induced current i is as shown in the figure 7, determine the direction of $\vec{B}$.
2. Determine the expression of i in terms of B, ℓ, R and v.
3. Show that the rod is under to the action of the Laplace's force $\vec{F}_e$ of expression $\vec{F}_e = -\frac{B^2\ell^2 v}{R}\vec{j}$.
4. a) Show that the differential equation that describes the variation of P as a function of time can be written as:

$$F = \frac{B^2\ell^2}{Rm}P + \frac{dP}{dt}.$$

b) Referring to part A, determine the solution of this differential equation.

c) Knowing that the rod reaches a limiting speed v_ℓ = 2 m/s after a time very close to 20 s, determine the values of R and F.

III- [7 pts] A nuclear generator for a pacemaker

Given: m($^{238}_{93}Np$) = 237.999791 u ; m($^{238}_{92}U$) = 238.000185 u; m($^{2}_{1}H$) = 2.013552 u; m($^{238}_{94}Pu$) = 237.997855 u ;
m($^{1}_{0}n$) = 1.008665 u; 1 u = 931.5 MeV/c^2 = 1.66×10^{-24}g; h = 6.626×10^{-34} SI; c = 3×10^8 m/s; m(e$^-$) = 9.1×10^{-31} kg;
1 eV = 1.602×10^{-19} J; 1 Ci = 3.7×10^{10} Bq.

Pacemakers are used to stimulate a regular heartbeat when the body's natural electrical pacing system does not function properly. A plutonium-238 ($^{238}_{94}Pu$) nucleus, an α-emitter, is synthesized by the first two of the three following successive nuclear reactions:

1- $^{2}_{1}H + ^{238}_{92}U \longrightarrow ^{238}_{93}Np + 2\,^{a}_{z}p$; 2- $^{238}_{93}Np \longrightarrow ^{238}_{94}Pu + ^{0}_{-1}e + \gamma + ^{0}_{0}\bar{\nu}$; 3- $^{238}_{94}Pu \longrightarrow ^{A}_{Z}X + ^{4}_{2}He + \gamma$.

A heart pacemaker contains a certain amount of $^{238}_{94}Pu$ of initial activity 2.5 Ci and whose half-life is 87.8 years. The energy released by each α-disintegration makes it possible for the pacemaker to produce electric energy.

1. Determine, specifying the laws used, the particle $^{a}_{z}p$ and the nucleus $^{A}_{Z}X$.
2. Determine, in MeV, the minimum kinetic energy of the deuteron, a deuterium nucleus, in order to make the first reaction possible. Suppose that the other nuclei and particles are at rest.
3. The figure 8 shows the most probable β^- disintegrations of Np-238 into Pu-238.
   a) What does the energy 1292.0 keV represent?
   b) The energy of each of the radiations γ_1, γ_2, γ_3, … is said to be quantized.
      i) What is meant by quantized energy?
      ii) What can you tell about the sum [E($^{0}_{0}\bar{\nu}$) + KE(β^-)]? Why?
      iii) KE(β^-) is thus not quantized. Why?

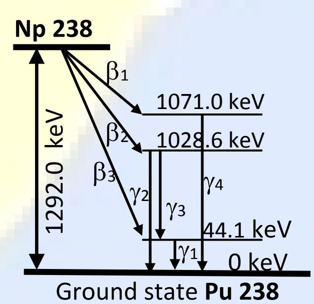


Fig. 8

c) Knowing that radiations γ_1 and γ_2 have respectively as wavelengths in vacuum $\lambda_1 = 2.814\times10^{-11}$ m and $\lambda_2 = 1.206\times10^{-12}$ m, deduce the wavelength of the radiation γ_3.

d) Referring to the energy diagram, determine the maximum speed of the particle β_3, of mass m, knowing that its kinetic energy is given by: $KE = (\gamma-1)mc^2$, with $\frac{1}{\gamma} = \sqrt{1-\frac{v^2}{c^2}}$.

4. a) In the third reaction, the frequency of the γ ray is 4.34×10^{17} Hz and the mass defect is $\Delta m_3 = 0.006$ u. Show that the kinetic energy of the α particle is equal to 8.95×10^{-13} J and that $E(\gamma)$ is negligible.

b) Determine the maximum power initially provided by this kind of nuclear generator.

5. a) Calculate the radioactive constant λ of a sample of ${}^{238}_{94}Pu$.

b) Calculate the mass m_0 of the plutonium 238 initially present in the pacemaker.

c) The pacemaker functions properly until its activity is reduced by 30% of its initial value. A pacemaker of this kind is used by a patient since 1974. Does this pacemaker still function properly in 2010?

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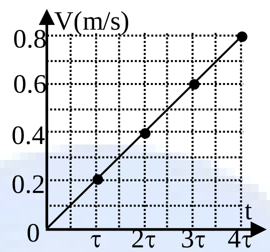
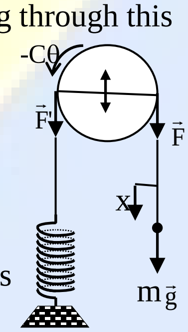
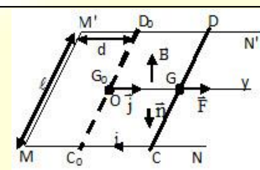
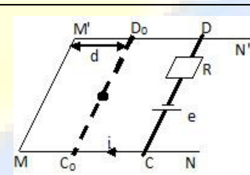
Entrance Exam 2010-2011 **<u>Solution of Physics</u>** **Duration: 2 hours**
04 July 2010

I- [6 pts] Torsion Pendulum

1.a	$V_2 = \dfrac{x_3 - x_1}{2\tau} = \dfrac{4.5 - 0.5}{2 \times 0.05}$ = 40 cm/s = 0.4 m/s.	1
1.b	(graph) 	1
1.c	Since v = v(t) is represented by a straight line, then v = at with a the slope of the straight line (also acceleration) ; with a = $\dfrac{v_4 - v_0}{4\tau} = \dfrac{0.8 - 0}{4 \times 0.05}$ = 4 m/s^2. But P = mv, then P = mat. Thus P = 0.2 t (SI)	2
1.d	According to Newton's second law: $\vec{F} + m\vec{g} = \dfrac{d\vec{P}}{dt}$, downward projection: $\dfrac{dP}{dt}$ = mg –F = ma $\Rightarrow$ F = mg – ma = m(g – a) $\Rightarrow$ F= 0.05(10 – 4) $\Rightarrow$ T = 0.3 N.	2
2	The wheel is subjected to its weight, the tension $\vec{F}'$ in the string and the reaction $\vec{N}$ of the axis of rotation. $\sum \text{moment} = \dfrac{d\sigma}{dt}$ $\Rightarrow$ 0 + 0 + F'R = $I_0\ddot{\theta}$; but a = $\ddot{x}$ = R$\ddot{\theta}$ $\Rightarrow$ $\ddot{\theta}$ = 0.2/(0.1×0.05) = 40 rad/s^2. I_0 = 0.3×0.1/40 $\Rightarrow$ $I_0 = 7.5\times10^{-4}$ kg·m^2.	2
3.a	The wheel is subjected in addition to the previous forces, to the tension $\vec{T}'_0$ of the spring with $T'_0 = T_0 = k\Delta\ell$. At equilibrium, $\sum \text{moment} = 0$; 0 + 0 – T'_0R + FR = 0 $\Rightarrow$ mgR = k$\Delta\ell$R$\Rightarrow$ $\Delta\ell$ = 0.05×10/5 = 0.1 m	2
3.b.i	x, abscissa of G, with respect to the equilibrium position. The horizontal plane passing through this equilibrium position is the gravitational potential energy reference level. $M_{wheel}gH + ½mv^2 + ½I_0\dot{\theta}^2 - mgx + ½C\theta^2 + ½k(\Delta\ell + x)^2$ = constant $\forall$ t. Deriving with respect to time, we obtain: $mR^2\dot{\theta}\,\ddot{\theta} + I_0\dot{\theta}\,\ddot{\theta} + C\theta\dot{\theta} + kR^2\theta\dot{\theta} = 0$ $\forall$ t. $\Rightarrow mR^2\ddot{\theta} + I_0\ddot{\theta} + C\theta + kR^2\theta = 0$ $\forall$ t. $\Rightarrow$ $\ddot{\theta} + \dfrac{kR^2 + C}{I_0 + mR^2}\theta = 0$  **Or :** For an abscissa x of G: x = Rθ and $\ddot{x}$ = R$\ddot{\theta}$ and the tension in the string becomes $\vec{F}'$ with: $\vec{F}' = -\vec{F} = k(\Delta\ell + x)\,\vec{j}$ $\Rightarrow$ The differential equation in θ ;	4

	$\sum \text{moment} = \frac{d\sigma}{dt}$: $0 + 0 - T'R + FR - C\theta = I_0\ddot{\theta}$; $-k(\Delta\ell + x)R + m(g - a)R - C\theta = I_0\ddot{\theta}$; $\Rightarrow I_0\ddot{\theta} = mgR - maR - k\Delta\ell R - kxR - C\theta$, with $x = R\theta$ et $a = R\ddot{\theta}$; $\Rightarrow I_0\ddot{\theta} = -mR^2\ddot{\theta} - kR^2\theta - C\theta$.	
ii	The differential equation is of the form: $\ddot{\theta} + \omega_0^2\theta = 0$; Then: $\omega_0^2 = \frac{kR^2 + C}{I_0 + mR^2} \Rightarrow T_0 = 2\pi\sqrt{\frac{I_0 + mR^2}{kR^2 + C}}$.	2
c	$T_0 \approx 2.95/4 = 0.74$ s $\Rightarrow 0.74^2 = 4\pi^2 \frac{I_0 + mR^2}{kR^2 + C}$; $kR^2 + C = \frac{4\pi^2}{0.74^2}(I_0 + mR^2)$ $C = 72.09(7.5\times10^{-4} + 0.05\times10^{-2}) - 5\times10^{-2} = 0.090 - 0.05 = 0.04$ SI.	2

II- [7 pts] Analogy

A.1.a	We have $E = u_{AC} = u_{AB} + u_{BC} = L\frac{di}{dt} + ri + Ri \Rightarrow E = L\frac{di}{dt} + (r + R)i$	1.5
A.1.b	For $t_0 = 0$, $i = 0 \Rightarrow b = -a$; $i = a - a\,e^{-\frac{t}{\tau}}$; $\frac{di}{dt} = a\frac{1}{\tau}e^{-\frac{t}{\tau}} \Rightarrow E = L\,a\frac{1}{\tau}e^{-\frac{t}{\tau}} + (R + r)a - (R + r)a\,e^{-\frac{t}{\tau}}$ Identifying: $E = (R + r)a \Rightarrow a = \frac{E}{R + r}$ et $L\frac{1}{\tau} = R + r \Rightarrow \tau = \frac{L}{R + r}$; and $i = \frac{E}{R + r}(1 - e^{-\frac{t}{\tau}})$.	2.5
A.1.c	The expression of the current I_0 in steady state is $I_0 = \frac{E}{R + r}$.	0.5
A.2	From figure 2, the value of I_0 is 5 mA (infinite time) and the value of the time-constant $\tau = 0.2$ ms, because for $t = \tau$, $i = 0.63I_0 = 3.15$ mA this gives $\tau = 0.2$ ms.	2
A.3	From figure 2, $I_0 = 5$ mA $= E/(R+r) \Rightarrow E = 5\times10^{-3}(r+R)$; From figure 3, $I_0 = 5$ A $= E/r \Rightarrow E = 5$ r. $\Rightarrow r + R = 1000\,r \Rightarrow r = 1000/999 \approx 1\Omega$ And $E \approx 5$ V and $L = \tau(R+r) \approx 0.2$ H.	2
B.1	The surface of the circuit increases therefore according to Lenz law, Laplace's force $\vec{F}_e$, is in the opposite direction to the displacement of the rod; using the right hand rule, $\vec{B}$ is so vertically upwards. 	1
B.2	According to the direction of i, $\vec{n}$ is vertically downwards. At the instant t, the magnetic flux of the field is: $\phi = -BS = -B\ell(y+d)$. The induced emf: $e = -\frac{d\phi}{dt} = +B\ell\frac{dy}{dt} = B\ell v$. $u_{DC} = 0 = Ri - e \Rightarrow i = \frac{e}{R} = \frac{B\ell v}{R}$. 	3.5

B.3	The magnitude of $\vec{F}_e$ is: $F_e = iB\ell\sin(\vec{\ell},\vec{B}) = \frac{B\ell v}{R}B\ell = \frac{B^2\ell^2 v}{R}$ $\vec{F}_e$ is in the opposite direction to $\vec{j}$, so $\vec{F}_e = -\frac{B^2\ell^2 v}{R}\vec{j}$.	1
B.4.a	According to Newton's second law: $\vec{F}+\vec{F}_e+m\vec{g}+\vec{N} = \frac{d\vec{P}}{dt}$; Projecting along $\vec{j}$, we obtain: $F - \frac{B^2\ell^2}{R}\frac{mv}{m} = \frac{dP}{dt}$; so $F = \frac{B^2\ell^2}{Rm}P + \frac{dP}{dt}$.	2
B.4.b	Referring to part A, the solution is in the form: $P = a' + b'e^{-\frac{t}{\tau'}}$. For t = 0, v = 0, then b' = - a'. In A: $E = L\frac{di}{dt} + (r + R)I$ and $a = \frac{E}{R+r}$ and $\tau = \frac{L}{R+r}$. Then: $a' = F/(\frac{B^2\ell^2}{Rm})$ and $\tau' = 1/(\frac{B^2\ell^2}{Rm})$; so $a' = \frac{F\,Rm}{B^2\ell^2}$ and $\tau' = \frac{Rm}{B^2\ell^2}$.	2
B.4.c	The limiting speed is reached for $t \approx 5\tau' = 20$ s $\Rightarrow \tau' = 4$ s. $P_{lim} = 0.1\times2 = 0.2$ kg·m/s $\Rightarrow R = \frac{B^2\ell^2\tau'}{m} = 0.09\times0.01\times4/(0.01) = 0.36\ \Omega$. $a' = P_{lim} = 0.01\times2 = 0.02$ kg·m/s $= F\cdot\tau' \Rightarrow F = a'/\tau' = 0.02/4 = 0.005$ N	3

III- [7 pts] A nuclear generator for a pacemaker

1	Conservation of the charge number: $1 + 92 = 93 + 2z \Rightarrow z = 0$ Conservation of mass number: $2 + 238 = 238 + 2a \Rightarrow a = 1 \Rightarrow {}^{a}_{z}p$ is a neutron ${}^{1}_{0}n$ Conservation of the charge number: $94 = Z + 2 \Rightarrow z = 92$, Conservation of mass number: $238 = A + 4 \Rightarrow A = 234 \Rightarrow {}^{A}_{Z}X$ is a ${}^{234}_{92}U$ nucleus.	2
2	$KE({}^{2}_{1}H) + [m({}^{2}_{1}H) + m({}^{238}_{92}U) - m({}^{238}_{Z}Np) - 2m({}^{1}_{0}n)]c^2 \geq 0 \Rightarrow$ $KE({}^{2}_{1}H) \geq [m({}^{238}_{93}Np) + 2m({}^{1}_{0}n) - m({}^{2}_{1}H) - m({}^{238}_{92}U)]c^2$ $KE({}^{2}_{1}H) \geq [237.999791 + 2\cdot1.008665 - 2.013552 - 238.000185]931.5 =$ $KE({}^{2}_{1}H) \geq 0.003384\times931.5 = 3.152$ MeV	2.5
3.a	It represents the energy released by β^--disintegration of neptunium	0.5
3.bi	An energy is said to be quantized, when it takes only well-defined values (discrete, discontinuous,..)	1
3.bii	The liberated energy is written: $E = \Delta m_2c^2 = KE(\beta^-) + E({}^{0}_{0}\bar{\nu}) + E(\gamma) =$ constant. Since $E(\gamma)$ is quantized then the sum $[E({}^{0}_{0}\bar{\nu}) + KE(\beta^-)]$ is quantized.	1.5
3.biii	$E({}^{0}_{0}\bar{\nu})$can take any value …$KE(\beta^-)$ then it is not quantized.	1
3.c	$E(\gamma_3) = E(\gamma_2) - E(\gamma_1) \Rightarrow \frac{1}{\lambda_3} = \frac{1}{\lambda_2} - \frac{1}{\lambda_1} \Rightarrow \lambda_3 = 1.26\times10^{-12}$ m.	2
3.d	$KE(max) = E - E(\gamma_3) = 1292.0 - 44.1 = 1247.9$ keV $1247.9\cdot1.602\times10^{-16} = 1.999\times10^{-13}$ J $(\gamma-1)9{,}1\times10^{-31}\cdot9\times10^{16} = 1.999\times10^{-13}$ J $\Rightarrow \gamma-1 = 2.441$ $\gamma = 3.441 \Rightarrow 0.0844 = 1-\frac{v^2}{c^2} \Rightarrow \frac{v^2}{c^2} = 0.156 \Rightarrow \frac{v^2}{c^2} = 0.9155 \Rightarrow \frac{v}{c} = 0.957$; $v = 0.957c = 2.871\times10^8$ m/s.	2
4.a	The energy of the photon γ : $E(\gamma) = h\nu = 6.626\times10^{-34}\cdot 4.34\times10^{17} = 2.876\times10^{-16}$ J . Energy released by the reaction of disintegration: $E_3 = \Delta m_3\cdot c^2 = 0.006\cdot931.5\cdot1.602\times10^{-13}$ $E_3 = 8.95\times10^{-13}$ J $KE(\alpha) = E_3 - E(\gamma) = E_3$ because $E(\gamma) << E_3 \Rightarrow KE(\alpha) = 8.95\times10^{-13}$ J.	2.5
4.b	The maximum power $P_m = A_0\times E_3$. $A_0 = 2.5\cdot3.7\times10^{10} = 9.25\times10^{10}$ Bq; $P_m = 9.25\times10^{10}\cdot8.95\times10^{-13} = 0.0828$ W.	1.5
5.a	The radioactive constant $\lambda = \ln(2)/(87.8\cdot365\cdot24\cdot3600) = 2.50\times10^{-10}$ s^{-1}.	1

5.b	$A_0 = \lambda \cdot N_0 \Rightarrow$ The initial number of nuclei: $N_0 = A_0/\lambda = \cdot 9.25\times10^{10}/2.5\times10^{-10} = 3.7\times10^{20}$ nuclei; The initial mass: $m_0 = 3.7\times10^{20}\cdot 238\cdot 1.66\times10^{-24} = 0.146$ g.	2
5.c	$A = A_0\, e^{-\lambda t}$; $0.7 = e^{-\lambda t} \Rightarrow \ln(0.7) = -\lambda t \Rightarrow t = 45.18$ years. 2010-1974 = 36 years (yes if there is no problem of circuit)	1.5