Entrance Exam 2009-2010 | **Physics** | **Time: 2 hours**
July 12, 2009

I- [20 pts] Determination of the characteristics (L, r) of a coil

In order to determine the characteristics r and L of a coil, we set up the circuit of the adjacent figure which is formed of the coil (L, r), a resistor of resistance R = 2.5 Ω, a generator of constant emf E = 6 V and of negligible resistance and a switch K. At the instant $t_0 = 0$, we close the switch K. At an instant t, the circuit carries a current i.

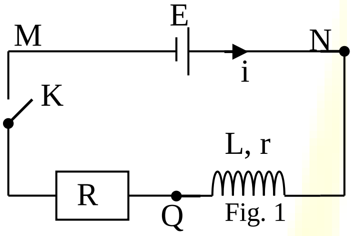

Fig. 1

1. a) Derive the differential equation describing the variations of i as a function of time.

   b) The solution of this equation is of the form: $i = I_0(1 - e^{-\frac{t}{\tau}})$, where I_0 and τ are constants.

   i) Determine, in terms of the given, the expressions of I_0 and τ.

   ii) Give, as a function of time, the expression of the voltage u_{MQ}.

2. The variations of the voltages u_{NQ} and u_{MQ}, as a function of time, are given in the adjacent waveform.

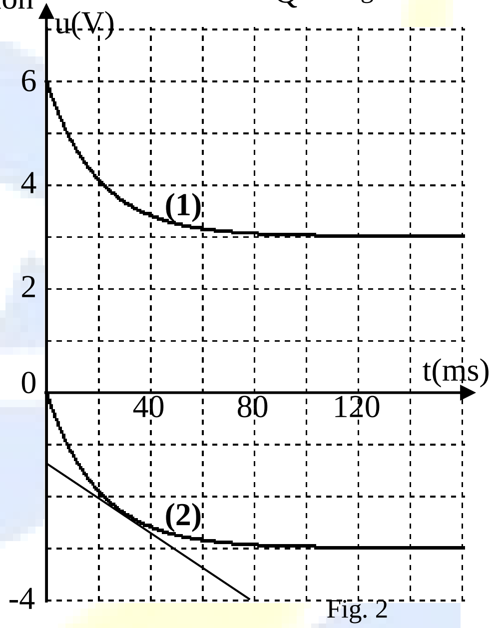

Fig. 2

   a) Specify, with justification, the curve which gives the variations of u_{MQ} as a function of time.

   b) Determine the value of r and that of I_0.

3. a) Show that the equation of the tangent to the curve (2) at a point of abscissa t' is given by:

$$u = -\frac{RI_0}{\tau} e^{-\frac{t'}{\tau}}(t - t') - RI_0(1 - e^{-\frac{t'}{\tau}}).$$

   b) i) Show that this tangent meets the asymptote to this curve at a point of abscissa t = t' + τ.

   ii) Deduce the value of τ and that of L.

4. To be sure of the values of r and L, we manage to obtain the voltage u_S, with $u_S = u_{NQ} + u_{MQ}$.

   a) Give the expression of u_S in terms of the given.

   b) Determine, using figure 3, the value of u_S at the instant t = τ.

   c) Deduce the value of L and that of r.

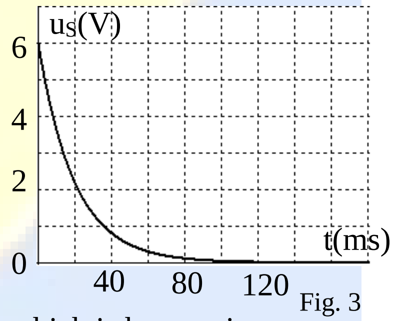

Fig. 3

II- [20 pts] The Plutonium

The fifteen known isotopes of plutonium are alpha emitter except the plutonium 241 which is beta emitter; the isotopes 239 and 241 are also fissile. Plutonium 238 has commercial and military applications.

Given: Part of the periodic table: $_{92}U$; $_{93}Np$; $_{94}Pu$; $_{95}Am$; $_{96}Cm$; $c = 2.9979\times10^{8}$ m/s;
1 MeV = 1.6022×10^{-13} J; 1 u = 1.66054×10^{-27} kg = 931.5 MeV/c^2; $m_n = 1.0087$ u; $m_P = 1.0073$ u.
Atomic molar mass of plutonium 238: M = 238 g·mol^{-1}; Avogadro's constant: $N_A = 6.022\times10^{23}$mol^{-1}.

A. The plutonium 238 α emitter

A heart pacemaker contains m = 130 mg of the isotope 238 of plutonium Pu whose half-life is $t_{1/2}$ = 87.8 years. The energy released by each disintegration allows the pacemaker to produce electric energy.

1. a) Specifying the laws of conservation used, write the equation of disintegration of a plutonium 238 nucleus.
   b) This reaction is accompanied by the emission of gamma radiation. Due to what is this emission?
2. a) Calculate the number N_0 of nuclei initially present in the pacemaker.
   b) Deduce the initial activity A_0 of this pacemaker.
   c) The pacemaker functions correctly as long as its activity remains higher or equal to 5.76×10^{10} Bq. Calculate the duration of its normal functioning under these conditions.

B. The fissile plutonium 239

Under the impact of a neutron, the plutonium 239 can undergo a fission nuclear reaction. The equation of this reaction is written as:

$${}^{1}_{0}n + {}^{239}_{94}Pu \longrightarrow {}^{135}_{52}Te + {}^{102}_{Z}Mo + y\,{}^{1}_{0}n$$

The adjacent table gives the binding energies E_b of the three nuclei.

Nuclei	${}^{239}_{94}Pu$	${}^{135}_{52}Te$	${}^{102}_{Z}Mo$
E_b (MeV)	1.79×10^3	1.12×10^3	8.64×10^2

1. Find the values of y and Z.
2. a) Write the expression giving the binding energy E_b of a ${}^{A}_{Z}X$ nucleus in terms of the mass defect and c.
   b) Give the expression of the mass of each of the above three nuclei in terms of its binding energy E_b, the mass m_n of a neutron, the mass m_P of a proton and c.
3. Determine, in MeV, the energy released by this reaction.
4. a) Calculate the binding energy per nucleon E_b/A for each of the three nuclei.
   b) Draw the shape of the Aston's curve that gives the variations of E_b/A as a function of A.
   c) Indicate the approximate location of the three nuclei on this curve.

III- [20 pts] Oscillator-Accelerometer

A- The horizontal oscillator

A horizontal oscillator, formed of an object of mass m = 2.5 mg (2.5×10^{-6} kg), is attached to two fixed points by means of two identical springs, each of natural length ℓ_0 and of stiffness k = 0.25 N/m. At equilibrium, the length of each spring is $\ell_0 + \Delta\ell$.

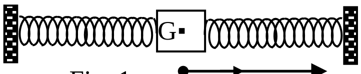

Fig. 1

The center of inertia G of the object can move on the horizontal axis (O; $\vec{i}$), O being the position of G at equilibrium.

The oscillator, initially at rest, is set in oscillation starting from the instant $t_0 = 0$. At an instant t, the position of G is located by its abscissa x. During motion, the springs remain always stretched and the forces of friction are negligible. (Fig. 1)

1. a) Show that, at the instant t, the elastic potential energy has the expression: $P.E_e = k\Delta\ell^2 + kx^2$.
   b) Derive the differential equation that describes the variations of x.
   c) Deduce the value of the proper angular frequency ω_0 of this oscillator.
2. Using an appropriate system, we record the variations of the algebraic value $\ddot{x}$ of the acceleration in terms of x (Fig. 2).
   a) Show that this graph is in agreement with the obtained differential equation.
   b) Determine, using the graph, the experimental value ω_{exp} of the proper angular frequency.

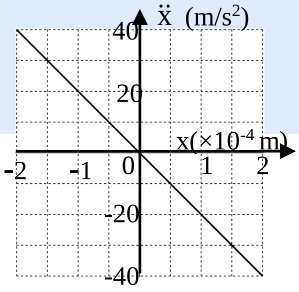

Fig. 2

B- Accelerometer

A truck, of mass 19 000 kg and moving with the speed 90 kmh^{-1}, is hit in the back by a car of mass 1500 kg moving with the speed 130 kmh^{-1}. The collision is completely inelastic (soft collision) and the two vehicles are equipped with airbags.

1. Show that the speed of the set formed of the two vehicles right after the collision is V = 25.8 m/s.
2. Knowing that the duration of the collision is 40 ms, determine:
   a) the acceleration, supposed constant, of each vehicle;
   b) the force that the truck exerts on the car.
3. The functioning of an airbag is controlled by an accelerometer (A), which is the oscillator already considered, able to detect any acceleration the vehicle is subjected to, i.e. any variation of capacitance ΔC of a grouping of suitably connected capacitors; the starting of (A) begins when ΔC, and consequently the value of the acceleration, exceeds a certain threshold ($\Delta C_{threshold} = 3\times10^{-12}$ F). The physical quantity ΔC is written in the form $\Delta C = 2\times10^{-4}\ x^2$, where x is the abscissa of G. The adjacent table gives estimation on the possible consequences of a collision on the passengers of a vehicle, each passenger having fastened his seat belt.
   a) What will the state of each passenger be?
   b) Tell, with justification, if the airbag will open.

acceleration	Estimation of possible effects on the passengers
100 m/s^2	Bearable for young people in good health
150 m/s^2	Risked internal bleeding with lesions
200 m/s^2	no chances of survival

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Entrance exam 2009-2010 **Solution of Physics** **Duration: 2 hours**
12 july 2009

I- [20 pts] Determination of the characteristics (L, R) of a coil

Part of the Q.	Correction	Mark
1.a	$u_{NM} = u_{NQ} + u_{QM}$; = $u_{NQ} = ri + L\frac{di}{dt}$; and $u_{QM} = Ri$. $\Rightarrow E = Ri + ri + L\frac{di}{dt}$; $\Rightarrow E = (R + r)i + L\frac{di}{dt}$	2.00
1.b.i	$\frac{di}{dt} = \frac{I_0}{\tau} e^{-\frac{t}{\tau}}$; $\Rightarrow E = (R + r) I_0(1- e^{-\frac{t}{\tau}}) + L\frac{I_0}{\tau} e^{-\frac{t}{\tau}}$. By identification: $\Rightarrow E = (R + r) I_0 \Rightarrow I_0 = \frac{E}{R+r}$ et $r+R = \frac{L}{\tau} \Rightarrow \tau = \frac{L}{r+R}$	3.00
1.b.ii	u_{MQ}. = - Ri = - $RI_0(1 - e^{-\frac{t}{\tau}})$	1.00
2.a	At the instant $t_0 = 0$, $(u_{MQ})_0 = 0$ and from 5τ $(u_{MQ})_{5\tau} = -RI_0 < 0$, it is the curve (2), or using ….	1.00
2.b	U_{NQ}.= $ri + L\frac{di}{dt}$, from 5τ $(u_{NQ})_{5\tau} = rI_0 = 3$ V; or $(u_{MQ})_{5\tau} = -3$ V $\Rightarrow rI_0 = -(- RI_0) \Rightarrow r = R = 2.5\ \Omega$ et $I_0 = \frac{E}{R+r} = \frac{6}{2.5+2.5} = 1.2$ A	2.50
3.a	The slope of the tangent is given by: $\frac{du_{MQ}}{dt} = -R\frac{di}{dt} = -\frac{RI_0}{\tau} e^{-\frac{t}{\tau}}$, At the instant t', it is equal to: $\frac{du_{MQ}}{dt} = -\frac{RI_0}{\tau} e^{-\frac{t'}{\tau}}$ u = at + b, equation of the tangent; $u = -\frac{RI_0}{\tau} e^{-\frac{t'}{\tau}} t + b$ For t = t': $u = -RI_0(1 - e^{-\frac{t'}{\tau}})$. Replacing: $-RI_0(1 - e^{-\frac{t'}{\tau}}) = -\frac{RI_0}{\tau} e^{-\frac{t'}{\tau}} t' + b$ Thus $b = \frac{RI_0}{\tau} e^{-\frac{t'}{\tau}} t' - RI_0(1 - e^{-\frac{t'}{\tau}}) \Rightarrow u = -\frac{RI_0}{\tau} e^{-\frac{t'}{\tau}} t + \frac{RI_0}{\tau} e^{-\frac{t'}{\tau}} t' - RI_0(1 - e^{-\frac{t'}{\tau}})$ And then: $u = -\frac{RI_0}{\tau} e^{-\frac{t'}{\tau}} (t - t') - RI_0(1- e^{-\frac{t'}{\tau}})$	2.50
3.b.i	For the asymptote, $u = -RI_0$; $\Rightarrow -RI_0 = -\frac{RI_0}{\tau} e^{-\frac{t'}{\tau}} (t - t') - RI_0(1- e^{-\frac{t'}{\tau}})$ $\Rightarrow 1 = \frac{1}{\tau} e^{-\frac{t'}{\tau}} (t - t') + 1 - e^{-\frac{t'}{\tau}} \Rightarrow \frac{1}{\tau} e^{-\frac{t'}{\tau}} (t - t') - e^{-\frac{t'}{\tau}} = 0$; $\Rightarrow t = t' + \tau$.	1.50
3.b.ii	From the graph τ = 20 ms = 0.02 s.	2.00

	But $\tau = \frac{L}{r+R} \Rightarrow L = \tau(r + R) = 0.02\times5 = 0.1$ H	
4.a	$u_S = u_{NQ} + u_{MQ} = - Ri + ri + L\frac{di}{dt}$; $\Rightarrow u_S = L\frac{di}{dt}$.	1.50
4.b	At the instant $t = \tau$, $u_S = 2.2$ V	1.00
4.c	From figure 2: à $t = \tau$, $i = 1.9/2.5 = 0.76$ A. $E = (R + r)i + L\frac{di}{dt} \Rightarrow E = \frac{L}{\tau}i + u_S$; then: $6 = \frac{L}{0.02}\times0.76 + 2.2$ $\Rightarrow L = 0.1$ H. Also: $6 = (2.5 + r)0.76 + 2.2 \Rightarrow r = 2.5\ \Omega$.	2.00
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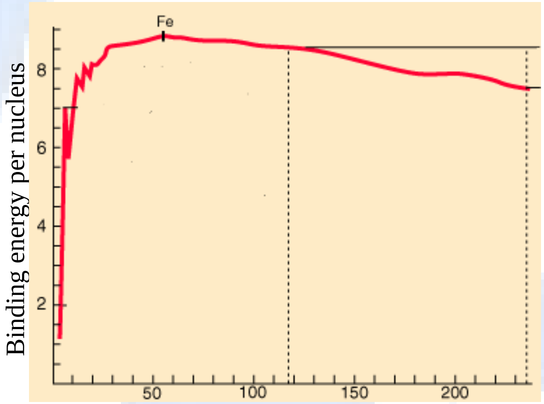
II- Le plutonium

Part of the Q.	Correction	Mark
A.1.a	${}^{238}_{94}Pu \longrightarrow {}^{A}_{Z}X + {}^{4}_{2}He$ Law of conservation of Z: $94 = Z + 2 \Rightarrow Z = 92$; Law of conservation of A: $238 = A + 4 \Rightarrow A = 234$. It is a uranium nucleus: ${}^{234}_{92}U$	2.00
A.1.b	The uranium nucleus is obtained in an excited state. The emission of γ is due to the decay of the daughter nucleus.	1.00
A.2.a	Formula, the number of nuclei $N_0 = (130\times10^{-3}/238)\times 6.,022\times10^{23} = 3.289\times10^{20}$ nuclei	1.50
A.2.b	The radioactive constant $\lambda = \ln2/t_{1/2} = 0.693/87.8\times365\times24\times3600) = 2.5\times10^{-10}\ s^{-1}$. The initial activity: $A_0 = \lambda N_0 = 2.5\times10^{-10}\times3.289\times10^{20} = 8.23\times10^{10}$ Bq	2.00
A.2.c	$A = A_0e^{-\lambda t}$; $\lambda t = \ln(A_0/A) = \ln(8.23\times10^{10}/5.76\times10^{10}) = 0.357$ $\Rightarrow t = 0.357/2.5\times10^{-10} = 1.43\times10^{9}$ s = 45.3 ans.	2.50
B.1	${}^{1}_{0}n + {}^{239}_{94}Pu \longrightarrow {}^{135}_{52}Te + {}^{102}_{Z}Mo + y\ {}^{1}_{0}n$ Law of conservation of Z: $94 = 52 + Z + 0 \Rightarrow Z = 40$; Law of conservation of A: $239 + 1 = 135 + 102 + y \Rightarrow y = 3$. ${}^{1}_{0}n + {}^{239}_{94}Pu \longrightarrow {}^{135}_{52}Te + {}^{102}_{42}Mo + 3\ {}^{1}_{0}n$	2.00
B.2.a	$E_\ell = [Zm_P + (A-Z)m_n – m_X]c^2$	1.00
B.2.b	$m({}^{239}_{94}Pu) = 94m_P + 145m_n - E_\ell(Pu)/c^2$; $m({}^{135}_{52}Te) = 52m_P + 83m_n - E_\ell(Te)/c^2$; $m({}^{102}_{42}Mo) = 42m_P + 60m_n - E_\ell(Mo)/c^2$	1.50
B.3	$E = \Delta mc^2 = [m({}^{1}_{0}n) + m({}^{239}_{94}Pu) - [m({}^{135}_{52}Te) + m({}^{102}_{40}Mo) + 3\ m({}^{1}_{0}n)]c^2$ $E = [(94 – 52 - 42)m_P + (1 + 145 - 83 – 60 - 3)\ m_n - E_\ell(Pu)/c^2 + E_\ell(Te)/c^2 + E_\ell(Mo)/c^2]c^2$ $E = E_\ell(Te) + E_\ell(Mo) - E_\ell(Pu)$	3.00

	$E = 1.12\times10^3 + 8.64\times10^2 - 1.79\times10^3 = 194$ MeV	
B.4.a	$\frac{E_\ell}{A}(Pu) = 1790/239 = 7.49$ MeV/nucleon; $\frac{E_\ell}{A}(Mo) = 864/102 = 8.47$ MeV/nucleon; $\frac{E_\ell}{A}(Te) = 1120/135 = 8.30$ MeV/nucleon;	1.50
B.4.b B.4.c	See figure 	2.00
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III- Oscillator-Accelerometer

Part of the Q.	Correction	Mark
A.1.a	$P.E_e = ½k(\ell-\ell_0)^2$; $E_{Pe} = ½k(\Delta\ell + x)^2 + ½k(\Delta\ell - x)^2 = k\Delta\ell^2 + kx^2$	1.50
A.1.b	Negligible friction, conservation of the mechanical oscillator: $ME = ½mv^2 + kx^2 = \text{constant}$. Derivative with respect to time: $mv\dot{v} + 2kx\dot{x} = 0; \Rightarrow m\ddot{x} + 2kx = 0$ and then we obtain the differential equation: $\ddot{x} + \frac{2k}{m}x = 0$	2.50
A.1.c	This equation is of the form $\ddot{x} + \omega_0^2 x = 0$, with $\omega_0^2 = \frac{2k}{m}$ $\Rightarrow \omega_0 = \sqrt{\frac{0.5}{2.5\times10^{-6}}} = 447.2$ rd/s	2.00
A.2.a	The curve is carried by a straight line of negative slope; this shows that $\ddot{x}$ is a linear function of x, i.e: $\ddot{x} = -\lambda x$ which is in agreement with the differential equation: $\ddot{x} = -\frac{2k}{m}x$	1.50
A.2.b	$-\lambda = \frac{\Delta\ddot{x}}{\Delta x} = -2\times10^5 = -\omega_{exp}^2 \Rightarrow \omega_{exp} = 447,2$ rd/s.	2.00
B.1	Conservation of linear momentum: $m_C\vec{V}_C + m_v\vec{V}_v = (m_C + m_v)\vec{V}'$ After projection: $m_CV_C + m_vV_v = (m_C + m_v)V \Rightarrow V = 25.83$ m/s	2.50
B.2.a	$a_C = \frac{\Delta V}{\Delta t}$ For the truck the acceleration: $a_C = \frac{\Delta V}{\Delta t} = \frac{25.83-25}{40\times10^{-3}} = 20.325\ m/s^2 \approx 20.3\ m/s^2$. For the car the acceleration $a_v = \frac{\Delta V}{\Delta t} = \frac{25.83-36.11}{40\times10^{-3}} = -257.5\ m/s^2$	2.50
B.2.b	Forces acting, $\sum\vec{F} = \frac{d\vec{P}}{dt}$; projection the force $F = \frac{\Delta P_v}{\Delta t} = m_v\frac{\Delta V_v}{\Delta t} = -3.86\times10^5$ N	2.00
B.3.a	We have: $\Delta C_{threshold} = 2\times10^{-4} x^2 = 3\times10^{-12} \Rightarrow x_{threshold} = 1.225\times10^{-4}$ m. Acceleration threshold: $a_{threshold} = \ddot{x}_{threshold} = -\omega_0^2 x = -2\times10^5\times 1.22\times10^{-4} = -24.5\ m/s^2$. For the driver of the truck, it may be that nothing arrives to him, while the driver of the car it runs a risk of death.	2.50

B.3.b	The airbag will open in the car because v > $a_{threshold}$. and nothing occurs in the truck	1.00
		20