LEBANESE UNIVERSITY
FACULTY OF ENGINEERING

الجامعة اللبنانية
كلية الهندسة

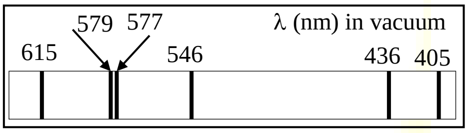
Entrance Exam 2008-2009 **Physics** **Duration: 2 h**

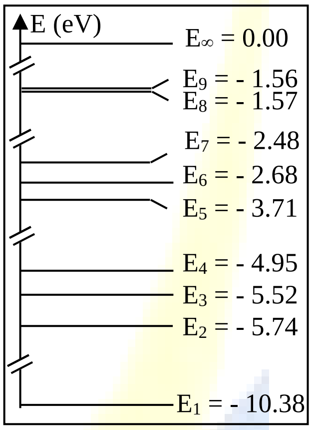
I- [16 pts] The mercury atom

Planck's constant = 6.626×10^{-34} Js, $c = 2.998 \times 10^{8}$ m/s, $e = 1.602 \times 10^{-19}$ C

A- Transitions

The adjacent figure shows a part of the emission spectrum of a mercury vapor lamp (M) and the energetic diagram of some energy levels of the mercury atom.





1. The energy levels of the mercury atom are quantized. Justify.
2. Calculate the ionization energy of the atom when it is in each of the energy levels: "E_1" and "E_5".
3. A mercury atom is in the energy level "E_9". It undergoes a downward transition to the ground state thus passing successively by the levels "E_5" and "E_2".
   a) Determine the transition to which a radiation of wavelength 577 nm is associated.
   b) Justify then the presence of the yellow doublet (577, 579).
4. Explain what is likely to happen if a:
   a) moving electron of energy 6.00 eV collides with a mercury atom that is in the ground state;
   b) photon, of energy 6.00 eV, were to be incident on the atom that is in the ground state.

B- Planck's Constant

The yellow doublet will be considered as one radiation of wavelength 578 nm.
The lamp (M), equipped with several filters, illuminates the cesium cathode of a photoelectric cell of work function $W_S = 2.15$ eV. For each of the wavelengths, we measure the maximum kinetic energy KE of the emitted electrons (adjacent table).

λ (nm)	615	578	546	436	405
KE (eV)	0	0	0.18	0.75	0.97

1. Determine the expression of KE in terms of $1/\lambda$.
2. Determine the value of the Planck's constant.
3. a) Determine the wavelength threshold λ_S associated to the cesium.
   b) Interpret the presence of the zeros in the table.
4. a) The photoelectric effect shows evidence of an aspect of light. Which one?
   b) Another phenomenon shows evidence of the other aspect of light. Which one?

II-[21 pts] The RLC series circuit

Consider the electric circuit diagram of the adjacent figure that includes an ideal generator of e.m.f E, a capacitor of capacitance C = 20 μF, a coil of inductance L and of negligible resistance, two resistors, ($R_1 = 5\ \Omega$ and $R_2 = 35\ \Omega$), and two switches K and K'. A suitable device is used to display the voltage $u_C = u_{AM}$ and the voltage $u_R = u_{BM}$.

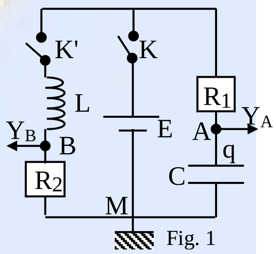


Fig. 1

A- Charging the capacitor

Initially, the two switches are opened and the capacitor is uncharged. At an instant $t_0 = 0$, we close K. At an instant t, the circuit carries a current i.

1. Redraw the circuit diagram and indicate the real direction of i.
2. Derive the differential equation that governs the variations of u_C as a function of time.
3. The solution of this differential equation is of the form: $u_C = A\,(1 - e^{-\frac{t}{\tau}})$. Determine the expressions of A and τ.
4. a) Using the waveform of figure 2, determine the values of E and τ.
   b) Using calculations, justify the value of τ.

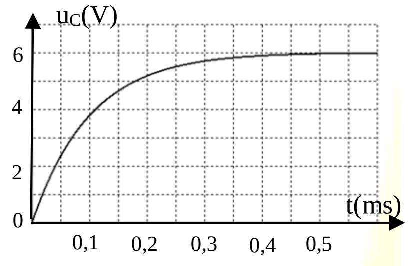


Fig. 2

B-Discharging the capacitor

When the capacitor is charged, we open K, then, at an instant $t_0 = 0$, K' is closed. The circuit is then the seat of electric oscillations, the duration of one oscillation being T. Starting from the instant $t_0 = 0$, we display the variations of the voltages u_C and u_R using respectively the scales 1 V/div and 0.2 V/div.

1. Knowing that the differential equation that governs the variations of u_C is of the form:

$$\frac{d^2u_C}{dt^2} + 2b\frac{du_C}{dt} + \omega_0^2 u_C = 0.$$

Find, in terms of R_1, R_2, L and C, the expressions of b and ω_0.

2. The solution of this differential equation is of the form: $u_C = Ae^{-25t}\cos(249\,t – 0{,}1)$. Determine the value of A.
3. a) Calculate the electric energy of the oscillator respectively at the instant $t_0 = 0$ and at the instant $t_1 = T$.
   b) Determine the value of the energy dissipated between the instants $t_0 = 0$ and $t_1 = T$.
   c) Calculate the average value of the energy dissipated between the instants $t_0 = 0$ and $t' = T/4$.
   d) Deduce the magnetic energy stored in the coil at the instant $t' = T/4$.
   e) Determine the value of the current at the instant $t' = T/4$.
   f) Determine the value of L.
4. The value of L may be obtained using another method. Determine its value.

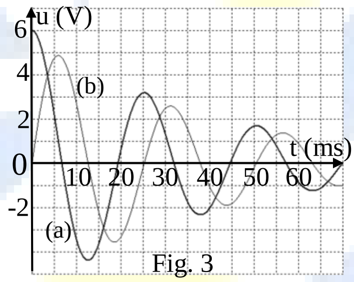


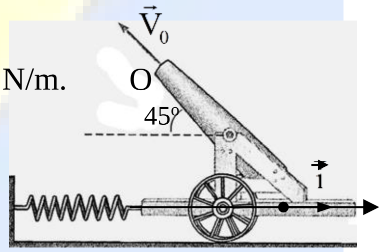
Fig. 3

III- [23 pts] Mechanics

A cannon is rigidly attached to a carriage, which can move without friction along horizontal rails. The carriage is connected to a post by a spring of stiffness $k = 2\times10^4$ N/m. A shell, considered as a particle of mass m = 200 kg, is launched from a point O (see figure) at the instant $t_0 = 0$ with a velocity $\vec{V}_0$ making an angle $\alpha = 45^o$ with the horizontal; it rises and reaches the maximum height h = 250 m.

The gravitational potential energy reference is the horizontal plane containing O.

Neglect air resistance and take $g = 10\ m/s^2$.



A-Motion of the shell

1. Using Newton's second law, determine in terms of α and V_0 (value of $\vec{V}_0$), the expression giving the value V_1 of the velocity of the shell in its highest position.
2. Determine the value of V_0.

B-Motion of the system (C) (canon, carriage)

- Theoretical study

Starting from the instant $t_0 = 0$, the system (C) of mass M = 5000 kg, launched with the horizontal velocity $\vec{V}_C = V_C\,\vec{i}$, performs a rectilinear sinusoidal motion of amplitude 1.42 m and of proper period T_0.

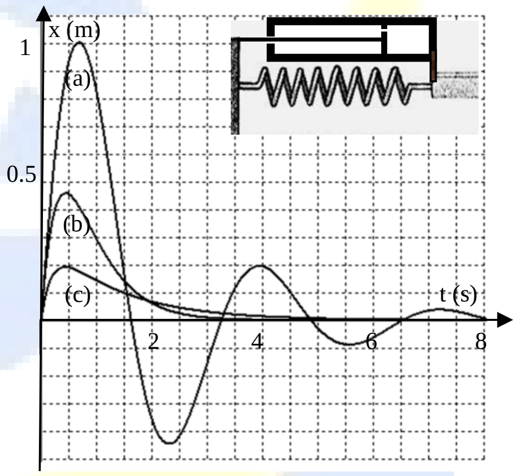
1. Applying the conservation of mechanical energy, determine the value of V_C.
2. Derive the differential equation that governs the motion of the system [(C), spring].
3. Determine the time equation of motion of (C).
4. Draw the shape of the curve giving the variations of the abscissa x of (C) as a function of time.
5. By comparing the linear momentum just before launching and that just after launching:
   a) show the non-conservation of the linear momentum of the system [(C), shell];
   b) applying Newton's second law, verify the value of V_C.

- Practical study

In fact, (C) is fitted with a shock absorber that exerts on (C) a damping force $\vec{F}$, with $\vec{F} = -\lambda\,\vec{v} = -\lambda\, v\,\vec{i}$, where λ is a positive constant and $\vec{v}$ the velocity of (C) at an instant t.

The adjacent figure presents three curves giving the variations of the abscissa x of (C) as a function of time, each for a given value of λ.

The considered values of λ are : $\lambda_1 = 5\times10^3$ kg/s, $\lambda_2 = 1.5\times10^4$ kg/s and $\lambda_3 = 3\times10^4$ kg/s.



1. For the better functioning of the cannon, it is necessary to ensure the "critical mode". The critical mode is the best for the fastest return of (C) to rest. What is then the corresponding value of λ?
2. Specify, for each of the two other curves, the corresponding mode.
3. What is the duration T of one oscillation during damped oscillations?
4. Compare T and T_0 and justify the result.

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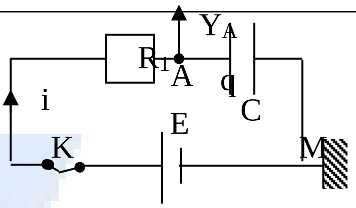
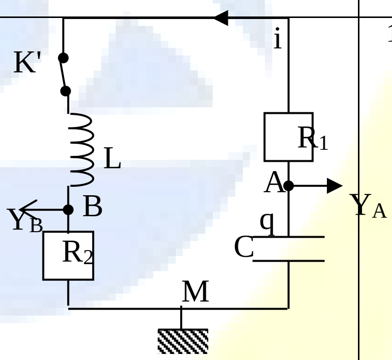
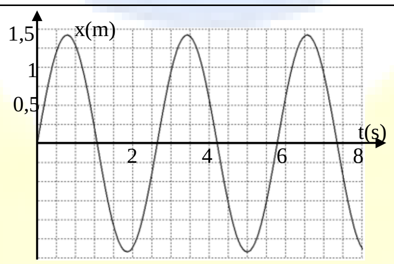
Entrance Exam 2008-2009 **Solution of Physics** **Duration: 2 h**

Part of the Q	Answer (First exercise Mercury atom)	Mark
A-1	The energy levels of the mercury atom are quantized because the energy of this atom been worth discrete has (this atom emits a discontinuous spectrum).	1
A-2	**For E_1** : $E_i = E_\infty - E_1 = 10.38$ eV ; For E_5 : $E_i = E_\infty - E_5 =$ **3.71 eV.**	0.5-0.5-0.5
A-3.a	$\Delta E = \frac{hc}{\lambda} = \frac{6.626\times10^{-34}\times2.998\times10^{8}}{577\times10^{-9}\times1.602\times10^{-19}} =$ **2.15 eV** This corresponds for a transition from the level E_9 to the level E_5.	0.5- 1 1
A-3.b	The yellow doublet (577) : from E_9 to E_5 and (579) : from E_8 to E_5. $\frac{6.626\times10^{-34}\times2.998\times10^{8}}{(-1.57+3.71)\times1.602\times10^{-19}} = 579$ nm	0.5 0.5-1
A-4.a	In the case of one electron, we have : $E + E_1 = -4.38$ eV, therefore the atom can pass one of the levels E_2, E_3 or E_4, and the rest energy will be taken by the electron	0.5 1-0.5
A-4.b	In the case of one photon: the atom remains in the fundamental state because $E + E_1 =$ **- 4.38 eV** , which does not correspond to any energy level of this atom	0.5
B-1	According to Einstein hypothesis : $KE = h\nu - W_0 \Rightarrow$ **$KE = \frac{hc}{\lambda} - W_0$**	1
B-2	KE is a linear function of $\frac{1}{\lambda}$, this implies that the slope: $hc = \frac{\Delta KE}{\Delta\left(\frac{1}{\lambda}\right)}$ $hc = \frac{(0.97-0.18)\times1.602\times10^{-19}}{\left(\frac{10^9}{405}-\frac{10^9}{546}\right)} = 1.985\times10^{-25}$ $\Rightarrow$ **$h = 6.62\times10^{-34}$ Js.**	1 1
B-3.a	$W_0 = \frac{hc}{\lambda_0} \Rightarrow \lambda_0 = \frac{hc}{W_0} = \frac{6.626\times10^{-34}\times2.998\times10^{8}}{2.15\times1.602\times10^{-19}} =$ **576 nm**	0.5- 1
B-3.b	615 and 578 nm are $> \lambda_0 \Rightarrow$ **KE = 0.**	1
B- 4.a	Particle aspect	0.5
B- 4.b	Interference – Diffraction	0.5

Part of the Q	*Answer (Second Exercise The RLC circuit)*	Mark
A-1	***1.*** *See the figure* 	1
A-2	$E = R_1 i + u_C$; *But* $i = \frac{dq}{dt} = C\frac{du_C}{dt} \Rightarrow E = R_1 C\frac{du_C}{dt} + u_C$.	0.5- 1- 0.5
A-3	$\frac{du_C}{dt} = \frac{A}{\tau}e^{-\frac{t}{\tau}} \Rightarrow E = R_1 C\frac{A}{\tau}e^{-\frac{t}{\tau}} + A - Ae^{-\frac{t}{\tau}}$; $\Rightarrow A = E$ *et* $(R_1 C\frac{A}{\tau} - A)e^{-\frac{t}{\tau}} = 0$ $\Rightarrow \boldsymbol{\tau = R_1 C}$.	0.5-0.5 0.5-0.5
A-4.a	*From the waveform :* $E = 6$ V *And for* $t = \tau$, $u_C = 0.63\times 6 = 3.78$ V $= 0.1$ ms $\Rightarrow \boldsymbol{\tau = 0.1}$ **ms**	0.5-0.5 0.5-0.5
A-4.b	*The value of* $\tau = R_1 C = 5\times 20\times 10^{-6} = 10^{-4}$ s $=$ **0.1 ms**	1
B-1	*We have:* $u_C = (R_1+R_2)\,i + L\frac{di}{dt}$; *with* $i = -C\frac{du_C}{dt}$ $u_C = -(R_1+R_2)\,C\frac{du_C}{dt} - LC\frac{d^2u_C}{dt^2}$ $\Rightarrow LC\frac{d^2u_C}{dt^2} + (R_1+R_2)C\frac{du_C}{dt} + u_C = 0$; $\Rightarrow \frac{d^2u_C}{dt^2} + \frac{R_1+R_2}{L}\frac{du_C}{dt} + \frac{1}{LC}u_C = 0$. *By comparison:* $\frac{d^2u_C}{dt^2} + 2b\frac{du_C}{dt} + \omega_0^2 u_C = 0 \Rightarrow b = \frac{R_1+R_2}{2L}$ *and* $\omega_0 = \sqrt{\frac{1}{LC}}$. 	1-0.5 1.5 0.5-0.5
B-2	*For* $t_0 = 0$, $u_C(0) = A\cos(-0.1) = 6 \Rightarrow$ $\boldsymbol{A = 6.03}$ **V.**	1
B-3.a	*At the instant* $t_0 = 0$: $u_C(0) = 6$ V *and* $W_E(0) = \frac{1}{2}C\,u_C^2(0) = 3.6\times 10^{-4}$ J *At the instant* $t_1 = T$: $u_C(T) = 3.2$ V *and* $W_E(T) = \frac{1}{2}C\,u_1^2(T) = \boldsymbol{1.024\times 10^{-4}}$ **J.**	0.5-0.5 0.5-0.5
B-3.b	*The value of the energy between* t_0 *and* t_1 : $\boldsymbol{\Delta W_E = 2.576\times 10^{-4}}$ **J.**	0.5
B-3.c	*The average energy between* t_0 *and* t' :. $\frac{\Delta W_E}{4} = \frac{2.956\times 10^{-4}}{4} = \boldsymbol{0.644\times 10^{-4}}$ **J.**	0.5
B-3.d	*At the instant* $t' = \frac{T}{4}$: $u_C = 0$ $W_E(t')=0$, *then* $W_m = W_E(0) - \frac{\Delta W_E}{4} = \boldsymbol{2.956\times 10^{-4}}$**J.**	0.5-0.5 0.5

B-3.e	*The intensity of the current at the instant t'* = $\frac{T}{4}$: $\frac{u_{R(t')}}{R} = \frac{(4.8\times0.2)}{35}$ = ***0.0274 A***	0.5-0.5 0.5
B-3.f	*But* $W_m = \frac{1}{2} Li^2 \Rightarrow L = \frac{2W_m}{i^2}$ = *0.79 H.*	0.5-0.5
4	$25 = \frac{R_1+R_2}{2L} = \frac{5+35}{2L} \Rightarrow L = \frac{40}{50} \Rightarrow$ ***L = 0.8 H;*** *OR* $T \approx T_0 = 2\pi\sqrt{LC} \Rightarrow L = \frac{T^2}{4\pi^2 C} = \frac{(0.025)^2}{4\pi^2\times2\times10^{-5}} \Rightarrow$ ***L = 0.79 H***	0.5 – 0.5

Part of the Q	Answer (third Exercise Mechanics)	Mark
A-1	Newton's second law: $m\vec{g} = \frac{d\vec{P}}{dt} \Rightarrow \frac{dP_x}{dt} = 0 \Rightarrow mV_{0x} = mV_1$ (because $V_{1y} = 0$), $\Rightarrow V_1 = V_{0x} = V_0 \cos\alpha$	0.5– 0.5 - 0.5- 0.5-0.5
A-2	Conservation of the mechanical energy: ME(O) = ME(1) $\Rightarrow 1/2\ m V_0^2 + 0 = 1/2\ m V_1^2 + mgh.$ $\Rightarrow 1/2\ m V_0^2 + 0 = 1/2\ m V_0^2 \cos^2\alpha + mgh \Rightarrow V_0^2 \sin^2\alpha + 0 = 2gh \Rightarrow V_0^2 = 4gh = 10^4$ $\Rightarrow V_0 = 100$ m/s.	1 1 0.5
B-ET-1	Conservation of the mechanical energy of the system [(C), spring] : $1/2 M V_C^2 + 0 = 0 + 1/2 k X_m^2$ $\Rightarrow V_C^2 = (k/M)\ X_m^2 = (2\times10^4/5000)\times1.42^2 = 8.06 \Rightarrow V_C = 2.84$ m/s.	0.5 0.5 0.5-0.5
B-ET -2	Conservation of the mechanical energy of the system [(C), spring] : $1/2Mv^2 + 1/2kx^2$ = constant. Derive with respect to time t: $Mv\dot{v} + kx\dot{x} = 0 \Rightarrow \ddot{x} + (k/M)\,x = 0$	0.5-0.5 1
B-ET -3	$x = a\sin(\omega_0 t + \varphi)$. at $t_0 = 0$, $x = 0 = a \sin\varphi \Rightarrow \varphi = 0$ or π. The velocity $v = \dot{x} = \omega_0 a\cos(\omega_0 t + \varphi)$. But at $t_0 = 0$, $\Rightarrow \dot{x}_0 = V_C = \omega_0 a\cos\varphi$, (a et $V_C > 0$) $\Rightarrow \varphi = 0\quad \omega_0^2 = k/m = 4 \Rightarrow \omega_0 = 2$ rd/s $\Rightarrow T_0 = 3{,}14$ s. $x = 1.42 \sin(2t)$ (x in m and t in s)	1.5 1 0.5-0.5
B-ET -4	4. (See figure).  	1
B-ET - 5.a	$\vec{P}_{before} = \vec{0}$ and $\vec{P}_{after} = M\vec{V}_C + m\vec{V}_0 \neq 0.$	0.5- 0.5-0.5
B-ET - 5.b	The system [(C), shell] is submitted to the normal reaction $\vec{N}$ (no friction) vertically upward, and the weight $(m+M)\vec{g}$ which is vertically downward, therefore: $\vec{N} + (m+M)\vec{g} = \frac{d\vec{P}}{dt} \Rightarrow \frac{dP_x}{dt} = 0 \Rightarrow \Delta P_x = 0 \Rightarrow mV_{0x} + MV_C = 0$ $\Rightarrow V_C = -\frac{m}{M} V_{0x} = -\frac{(200)\times(-100\cos 45^0)}{5000} \Rightarrow$ **V_C = 2.83 m/s**	0.5 0.5-0.5 0.5 0.5

B-EP-1	The corresponding value: (graph b) λ_2 **= 1.5×10^4 kg/s.**	1
B-EP-2	For λ_1 **= 5×10^3 kg/s**, Damped oscillations **(graph a)**; For λ_3 **= 3×10^4 kg/s** large damping (graph c).	1 1
B-EP-3	The duration T for one oscillation = **3.25 s**.	1
B-EP-4	$T > T_0$ this is because of the damping	0.5